Logic and Language, Proposition and Types, Proofs and Computation
The Particle Physics of Computer Science

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Introduction for 1st year students
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Programming Logic

- ProgLog group
- Professors: Thierry Coquand, Peter Dybjer, Bengt Nordström
- Permanent staff: Andreas Abel, Ana Bove, Nils Anders Danielsson, Ulf Norell
- Overall goal: Correctness of programs through logical means
  - Foundations of programming
  - Foundations of logics and mathematics
Correctness of programs

- Example: compiler correctness.
- Produced JVM byte code should faithfully represent Java source code.
  
  JAVA: \( y = x + 5 \)

  JVM: \( iload \ 1 \)
       \( bipush \ 5 \)
       \( iadd \)
       \( istore \ 2 \)

- There are infinitely many possible Java programs; we cannot test the compiler on all.
Compiler correctness

- Compiler is a function, inputs Java, outputs JVM.

  \[ \text{compile} : \text{Java} \rightarrow \text{JVM} \]

- Correctness means that compilation preserves meaning of code.
- Meaning of target: behavior of JVM code when run (executed in bytecode interpreter).
- Meaning of source: behavior of Java program when executed in an interpreter.

\[ \forall (p : \text{Java}) \rightarrow \text{interpret}(p) = \text{run}(\text{compile}(p)) \]
Compiler correctness

- What is a Java program, mathematically?
  - A sentence (string) following the Java grammar. [Languages, grammars, parsing]
  - Representable as abstract syntax tree. [Data structures, recursion]

- What is JVM code, mathematically?

- What is the meaning of a Java program? [Interpreter, semantics]

- What is the meaning of JVM code? [Machines, execution]

- What does “equal behavior” mean? [Relations, models of computation]

- How can we prove something for all Java programs? [Logic, induction]

- How can we be sure our proof is correct? [Proof theory, machine-assisted verification]
A simpler example

- Say we have a list \( l \) of natural numbers.

| \( i \) | 0 | 1 | 2 | \ldots | \( i \) | \ldots | \( n - 1 \) |
| \hline
| \( l \) | 2 | 3 | 5 | \ldots | lookup \( l \) \( i \) | \ldots | lookup \( l \) \( (n - 1) \) |
| incr \( l \) | 3 | 4 | 6 | \ldots | 1 + lookup \( l \) \( i \) | \ldots | 1 + lookup \( l \) \( (n - 1) \) |

- The following two should be equivalent.
  1. Making a copy of the list with each element increased by 1 (\texttt{incr}) and then taking the \( i \)th element (\texttt{lookup}).
  2. Taking the \( i \)th element (\texttt{lookup}) and increase it by 1 (\texttt{suc}).

\[ \forall (l : \text{List } \mathbb{N})(i : \mathbb{N}) \rightarrow \text{lookup } (\text{incr } l) \ i \equiv \text{suc } (\text{lookup } l \ i) \]
Modelling our example

- Data structures: natural numbers and lists
  [choice, composition, recursion]
- Functions: traversing a list
  [case distinction, recursion]
- Logic: proof of universal (\(\forall\)) statement
  [induction = case distinction + recursion]
Curry-Howard-Isomorphismus

Proposition $\equiv$ Set
proof $\equiv$ program/data

- Discovered in 1950s.
- Logic inspires programming language research.
- Programming language constructs find logical interpretations.
Particles of Computer Science

A logical approach to information and computation.

With quotes from L. & A. Wachowski,

*The Matrix Reloaded*
Causality (Implication)

**Merovingian:** You see, there is only one constant, one universal,
It is the only real truth: *causality.*
Action. Reaction.
Cause and effect.

Functions. Transforming input to output.
Implication. Conclusions from premises.

\[
\text{incr} : \text{List } \mathbb{N} \rightarrow \text{List } \mathbb{N}
\]
\[
\text{lookup-incr} : (l : \text{List } \mathbb{N})(i : \mathbb{N}) \rightarrow \text{lookup } (\text{incr } l) i \equiv \text{suc } (\text{lookup } l i)
\]
\[
(Even(n) \land \text{Prime}(n)) \rightarrow n \equiv 2
\]
Structure (Conjunction)

**Keymaker:** The system is based on the rules of a building. One system built on another. If one fails, all fail.

Tuples: several things put together.
E.g. the cons of lists, pairing head (1st element) and tail (rest).
Conjunction: 2 is an odd prime number.

\[(1, 2)\]
\[\text{head :: tail}\]
\[\text{Odd}(2) \land \text{Prime}(2)\]
Choice (Disjunction)

**The Oracle:** We can never see past the choices we don't understand.

**Morpheus:** Everything begins with choice.

**Neo:** Choice. The problem is choice.

Bits: false or true, zero or successor, empty list or cons. Each natural number is either even or odd.

```
b0 1
Bool false true
N zero suc
List [] _ :: _
```

\[ \text{Even}(n) \lor \text{Odd}(n) \]
Agent Jackson: You.
Smith: Yes me. Me, me, me!
Agent Jackson/Smith: Me too!

Smith: Go ahead, shoot. The best thing about being me—there’s so many me.

Recursive data types (e.g. lists).
Recursive functions (e.g. incr, lookup).
Recursive proofs (induction, e.g. lookup-incr).

\[(n : \mathbb{N}) :: (l : \text{List} \mathbb{N}) : \text{List} \mathbb{N}\]

\[\text{lookup} (n :: l) (\text{suc} i) = \text{lookup} / i\]
Agda

- Haskell-like programming language
- Based on the Curry-Howard-Isomorphismus
- Agda 2 developed at Chalmers since 2006
- Precursors since 1980s (ALF, Half, Alfa, Agda)
ProgLog Courses

DAT060 Logic in computer science (Coquand)
  • Proof calculi, applications of logic

TMV027 Finite automata and formal languages (Bove)
  • Grammars, parsing

DAT140 Types for proofs and programs (Dybjer)
  • Programming language theory
  • Type theory and Agda

TDA183 Models of computation (Nordström)
  • Lambda calculus, Turing machines
  • Undecidability