A Game-Theoretic Analysis of Strictly Competitive Multiagent Scenarios

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Alice, Bob, and Charlie raise hands

- Alice, Bob, and Charlie simultaneously decide whether to raise their hand or not
- Number of players that raise their hand is ...

  ...odd: Alice wins
  ...even and positive: Bob wins
  ...zero: Charlie wins
- What should Alice do?

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3	1
1	2

1	2
2	1

Outline

Ranking Games

Nash Equilibria in Ranking Games

Comparative Ratios: Nash Equilibria, Maximin Strategies, and Correlated Equilibria

Conclusions

Ranking Games

- A class of strategic (*i.e.*, normal-form) games
- A model for *strictly competitive multi*-agent situations
  - Parlor games
  - Competitive economic scenarios
  - Social choice settings
  - . . .
- Outcomes identified with rankings of the players
- Agents have preferences over ranks such that
  - higher ranks are weakly preferred
  - being first is strictly preferred over being last
  - agents are indifferent w.r.t. other agents' ranks

Ranking Games (More Formally)

- Definition: The *rank payoff* of player i is defined as a vector $r_i = (r_i^1, r_i^2, \ldots, r_i^n)$ such that
  - $r_i^k \geq r_i^{k+1}$ for all $1 \leq k \leq n-1$ and
  - $r_i^1 > r_i^n$
- For convenience, $r_i^1 = 1$ and $r_i^n = 0$
- Definition: A *ranking game* is a game where for any strategy profile $s \in S$ there is a permutation $(\pi_1, \pi_2, \ldots, \pi_n)$ of the players such that the payoff $p_i(s) = r_i^{\pi_i}$ for each player $i \in N$

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- *Binary* ranking games: $r_i^k \in \{0, 1\}$ for all i, k
- *Single-winner* games: $r_i = (1, 0, \ldots, 0)$ for all i

Nash Equilibria in Ranking Games

3	1
1	2

1	2
2	1

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3	1
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- Often very weak in ranking games (pure ones in particular)

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- Nash equilibrium: strategies are mutual best responses to each other
- Often very weak in ranking games (pure ones in particular)
- Quasi-strict Nash equilibrium (Harsanyi, 1973): *every* best response is played with positive probability

Nash Equilibria in Ranking Games

- Do all ranking games possess quasi-strict equilibria?

Nash Equilibria in Ranking Games

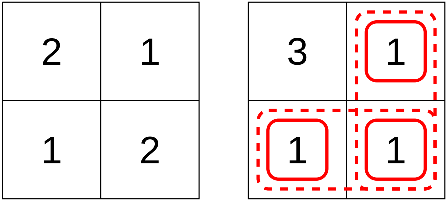
- Do all ranking games possess quasi-strict equilibria? No

2	1
1	2

3	1
1	1

Nash Equilibria in Ranking Games

- Do all ranking games possess quasi-strict equilibria? No



- It seems as if all *single-winner* games possess a *non-pure* equilibrium. Proven for:
  - Two-player ranking games (using a result by Norde, 1999)
  - $2 \times 2 \times 2$ single-winner games (nice combinatorial argument)
  - Single-winner games where at least two players have a positive security level

The Price of Cautiousness

3	1
1	2

1	2
2	1

- Nash equilibrium, quasi-strict Nash equilibrium

The Price of Cautiousness

$\frac{1}{2}$	3	1
$\frac{1}{2}$	1	2

1	2
2	1

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- Security level (maximin): guaranteed minimum payoff

The Price of Cautiousness

3	1
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- Nash equilibrium, quasi-strict Nash equilibrium
- Security level (maximin): guaranteed minimum payoff
- How much worse can a player be off when playing maximin instead of a Nash equilibrium?
- Price of cautiousness: Ratio between minimum payoff in a Nash equilibrium and (strictly positive) security level

The Price of Cautiousness in Ranking Games

Consider a game with at least 3 players, a player with k actions and strictly positive security level

- General ranking games: unbounded (involves taking limits)
- Binary ranking games: k (also w.r.t. quasi-strict equilibria)
  - Positive security level, hence for every opponent action profile there is some action that guarantees positive payoff, *i.e.*, payoff 1 in binary ranking games
  - Randomization over all k actions guarantees payoff $1/k$

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  - Lower bound

$\frac{1}{2}$	2	1
$\frac{1}{2}$	1	2

3	1
1	1

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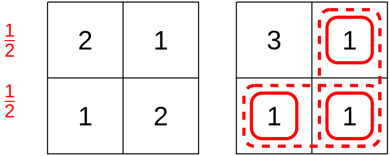

- Single-winner games, w.r.t. quasi-strict equilibria: $k - 1$

The Value of Correlation

3	1
1	2

1	2
2	1

- Correlated equilibrium: actions drawn according to joint distribution, no player can gain by deviating

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- Correlated equilibrium: actions drawn according to joint distribution, no player can gain by deviating
- Value of correlation (Ashlagi et al., 2005): By how much can correlation improve *social welfare*?
  - Mediation value: Ratio between maximum social welfare in correlated vs. Nash equilibrium
  - Enforcement value: Ratio of maximum social welfare in any outcome vs. correlated equilibrium

The Value of Correlation in Ranking Games

Consider a game with n players

- Symmetric rank payoffs: identical social welfare in every outcome, both mediation and enforcement value are 1
- Mediation value: $n - 1$
  - Upper bound is trivial
  - Lower bound

$(1, 1, 0)$	$(1, 0, 0)$
$(0, 1, 0)$	$(0, 1, 1)$

$(0, 1, 1)$	$(0, 1, 0)$
$(1, 0, 0)$	$(1, 1, 0)$

$(1, 0, 0)$	$(0, 0, 1)$
$(0, 0, 1)$	$(1, 0, 0)$

- Enforcement value: $n - 1$

Conclusions

- Ranking games: a model for *strict competitiveness* in the *multi*-agent case
- Nash equilibrium solutions: often very weak
- Maximin
  - Guarantees a certain payoff against indifferent (even irrational) opponents
  - Limited price of cautiousness (if there are few actions)
- Correlated equilibrium
  - Substantial increase in social welfare possible in scenarios with many players and asymmetric preferences over ranks
- Computational aspect
  - Maximin strategies and correlated equilibria computable in polynomial time
  - Nash equilibria just as hard to compute as in general games (Brandt et al., 2006)

Thank you for your attention!