Sum of Us

Strategyproof Selection from the Selectors

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Dagstuhl Seminar on
Computational Foundations of Social Choice

The Problem

- Approval voting
  - each voter approves of set of candidates (of any size)
  - choose candidate (or committee of desired size) with largest number of votes
- Strategyproof (assuming dichotomous preferences)

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- No longer the case when sets of candidates and voters coincide
  - scientific organizations (GTS, AMS, IEEE, IFAAMAS)
  - web graph, (directed) social networks, reputation systems

Outline

Sum of Us

- Set $N = [n]$ of agents
- Directed graph $G = (N, E) \in \mathcal{G}$, no self-loops
- Goal: select $S \in \mathcal{S}_k = \{T \subseteq N : |T| = k\}$ to maximize
  $\sum_{i \in S} \deg(i) = \sum_{i \in S} |\{j \in N : (j, i) \in E\}|$
- Mechanism $M : \mathcal{G} \to \Delta(\mathcal{S}_k)$
- Strategyproofness: probability of selecting i independent of edges (i, j) for $j \in N$

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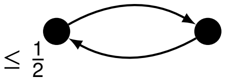

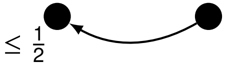

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- α-efficiency: for every graph,

$$\frac{\max_{S \in \mathcal{S}_k} \sum_{i \in S} \deg(i)}{\mathbb{E}_{S \sim M}\left[\sum_{i \in S} \deg(i)\right]} \le \alpha$$

Bad News

Theorem: Let $n \geq 2$, $k \leq n - 1$. Then there is no strategyproof and α-efficient deterministic mechanism for any finite α.

Proof: ...

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Cannot make sure only agent with any votes is selected (particularly surprising for $k = 1$, $k = n - 1$)

Proof

- Assume for contradiction M was such a mechanism
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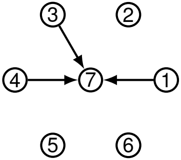
- Restrict domain to stars with n at the center



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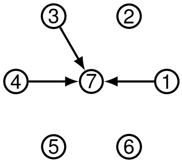

- Isomorphic to $\{0,1\}^{n-1}$, so we now look at mechanisms $M : \{0,1\}^{n-1} \to S_k$

Proof

(1) $n \notin M(\mathbf{0})$ (by assumption)

(2) $n \in M(x)$ for all $x \in \{0,1\}^{n-1} \setminus \{\mathbf{0}\}$ (by α-efficiency for finite α)

(3) $i \in M(x)$ iff $i \in M(x + e_i)$ for all $x \in \{0,1\}^{n-1}$ and $i \in N \setminus \{n\}$ (by strategyproofness)

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$$\sum_{x \in \{0,1\}^{n-1}} |M(x)| = \sum_{i \in N} |\{x \in \{0,1\}^{n-1} : i \in M(x)\}|$$

$$= \underbrace{(2^{n-1} - 1)}_{\text{by (1) and (2)}} + \textstyle\sum_{i \in N \setminus \{n\}} |\{x \in \{0,1\}^{n-1} : i \in M(x)\}|$$

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$$\overbrace{\sum_{x \in \{0,1\}^{n-1}} |M(x)|}^{2^{n-1}k \text{ (even)}} = \sum_{i \in N} |\{x \in \{0,1\}^{n-1} : i \in M(x)\}|$$

$$= \underbrace{(2^{n-1} - 1)}_{\text{odd}} + \underbrace{\textstyle\sum_{i \in N \setminus \{n\}} |\{x \in \{0,1\}^{n-1} : i \in M(x)\}|}_{\text{even by (3)}}$$

Random Partitions

Random m-partition (m-RP)

1. assign each agent i.i.d. to one of m sets
2. from each subset, select $\sim k/m$ agents with largest indegrees based on edges from *other* subsets

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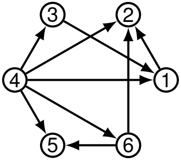

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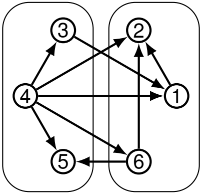

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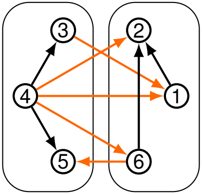

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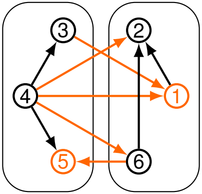

Bounds for Randomized Mechanisms

Theorem: *m*-RP is (universally) strategyproof for all n, k, m and

- 4-efficient (even) for $m = 2$,
- $1 + O(1/k^{\frac{1}{3}})$-efficient for $m \sim k^{\frac{1}{3}}$.

Theorem: Let $n \geq 2$, $k \leq n - 1$. Then there is no strategyproof and α-efficient mechanism for $\alpha < 1 + \Omega(1/k^2)$.

Bounds for Group-Strategyproof Mechanisms

- Group-strategyproofness: among any coalition of manipulators, some member does not gain
- Selecting a random k-subset is group strategyproof and n/k-efficient

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Theorem: There is no mechanism that is group-strategyproof and α-efficient for $\alpha < (n-1)/k$.

Homework

- Gap for randomized mechanisms
  $k = 1$: lower bound 2, upper bound 4

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- Gap for randomized mechanisms
  $k = 1$: lower bound 2, upper bound 4
- A mechanism selecting *one or two* agents:
  1. Fix any ordering of the agents
  2. Pick agent with “first” incoming edge from left to right, and agent with “first” incoming edge from right to left

Thank you!