

It is shown that the theory VNC_*^1 , introduced by the author in the companion paper [1], can formalize the construction of a sorting network of logarithmic depth, assuming that the existence of a suitable class of expander graphs can be proved therein.

It follows that under this assumption, there exist polynomial size boolean threshold formulas, s.t. their properties have polynomial size Frege proofs. This in turn implies – using a result of Atserias et al. [2] – that Frege systems can be polynomially simulated by the monotone sequent calculus MLK , a variant of the classical propositional sequent calculus LK without a symbol and rules for negation. A quasipolynomial simulation of Frege systems by MLK has been shown unconditionally by Atserias et al. [2].

References

- [1] E. Jeřábek, On theories of bounded arithmetic for NC^1 , *Ann. Pure Appl. Logic* **162** (2011), no. 4, 322–340. MR2747052
- [2] A. Atserias, N. Galesi and P. Pudlák, Monotone simulations of non-monotone proofs, *J. Comput. System Sci.* **65** (2002), no. 4, 626–638. MR1964646 (2005a:03115)