

Separation of Two Inseparable Logics



Separation of Test-Free Propositional Dynamic Logics over Context-Free Languages

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Motivation—Where are we?

Branching Time Logic

ω -Tree-Automata

PDL₀[CFL]



PDL₀[DCFL]



PDL₀[VPL]



nondeterministic
visibly pushdown



PDL₀[REG]



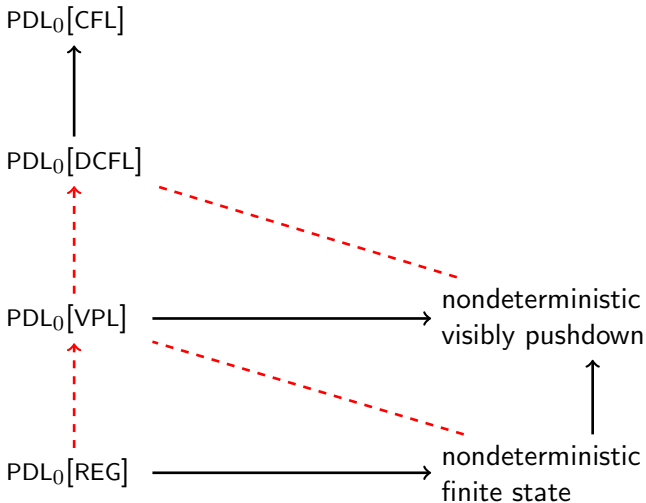
nondeterministic
finite state



Motivation—Where are we?

Branching Time Logic

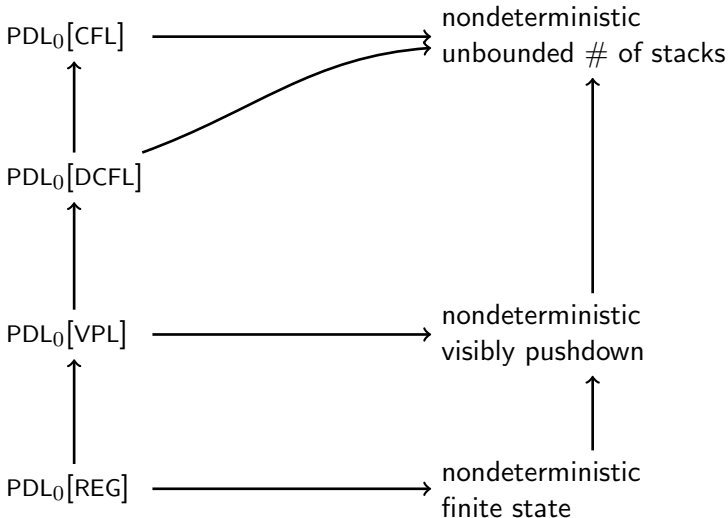
ω -Tree-Automata



Motivation—Where are we?

Branching Time Logic

ω -Tree-Automata



Motivation—What is our Target?

Branching Time Logic

ω -Tree-Automata

PDL₀[CFL]

PDL₀[DCFL]

PDL₀[VPL]

PDL₀[REG]

nondeterministic
unbounded # of stacks

nondeterministic
visibly pushdown

nondeterministic
finite state

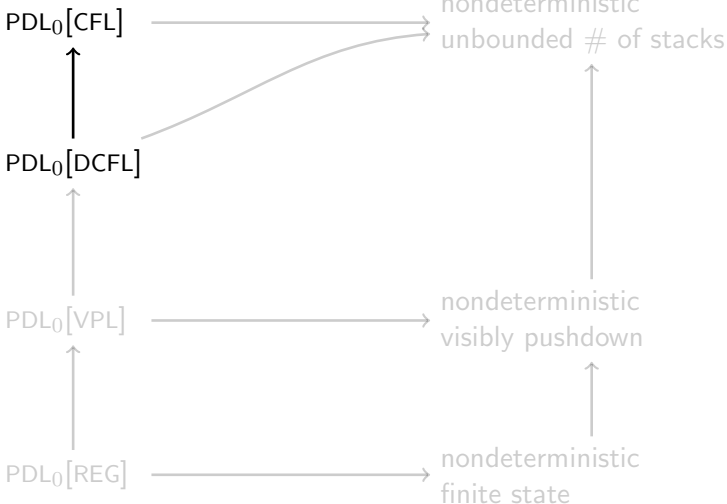


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- 3 Eliminations
- 4 Properties of Palindromes
- 5 Extraction

Syntax of $\text{PDL}_0[\cdot]$

[Fischer/Ladner '79], [Berman/Paterson '81] [Harel/(Pnueli/Stavi '83 | Raz '90)]
[Löding/Lutz/Serre '04], [Axelsson/Hague/Kreutzer/Lange/L. '10]

Let \mathfrak{A} be a class of languages $L \subseteq \Sigma^+$.

Definiton (Syntax of $\text{PDL}_0[\mathfrak{A}]$)

$$\varphi ::= \text{ff} \mid \text{tt} \mid p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \\ \text{EF}^L \varphi \mid \text{AG}^L \varphi$$

where $L \in \mathfrak{A}$, and p denotes a proposition.

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Test-free means

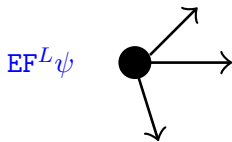
- ▶ no tests such as $\langle (\varphi? a)^* \rangle \psi$,
- ▶ no **EU/ER/AU/AR**-formulas.

Semantic of $\text{PDL}_0[\cdot]$

- ▶ Interpreted over a labeled transition system.

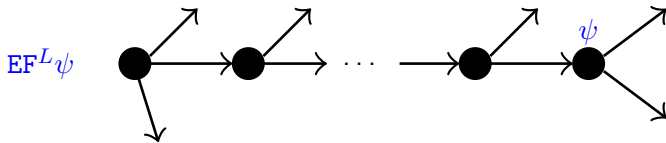
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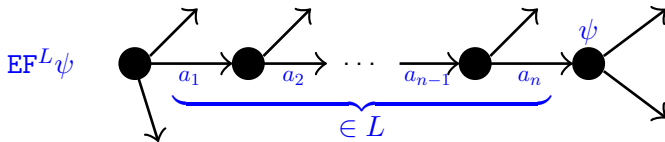
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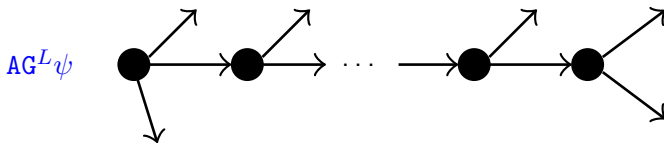
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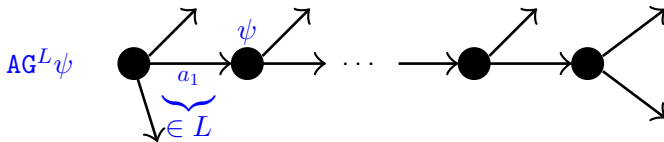
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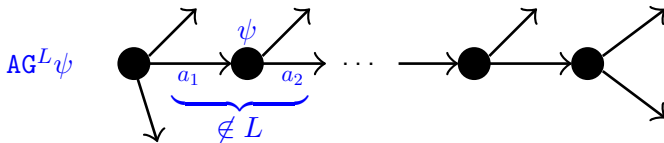
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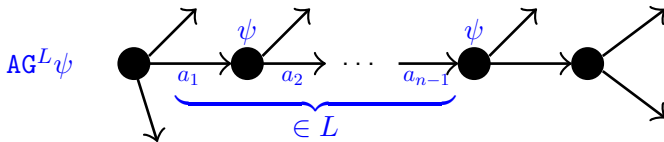
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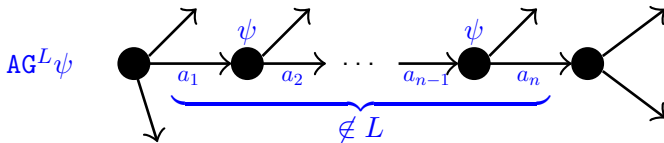
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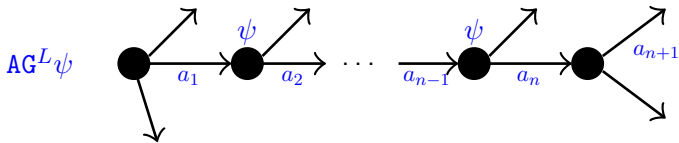
Semantic of $\text{PDL}_0[\cdot]$

- Interpreted over a labeled transition system.



Semantic of $\text{PDL}_0[\cdot]$

- Interpreted over a labeled transition system.



Example (Producer-Consumer Scenario)

- ▶ Producer can put items on a stack.
- ▶ Consumer can remove them and request further items.
- ▶ $\Sigma := \{p, c, r\}$.
- ▶ $L := \{w \in \Sigma^* \mid |w|_c = |w|_p \text{ and } |u|_c \leq |u|_p \text{ for any prefix } u \text{ of } w\}$.
- ▶ Specification:

$AG EX^p \text{tt}$ At any time it is possible to produce an object.

$AG^L (AX^c \text{ff} \wedge EX^r \text{tt})$ Whenever the buffer is empty, it is impossible to consume and possible to request.

$AG^{\bar{L}} (EX^c \text{tt} \wedge AX^r \text{ff})$ Whenever the buffer is non-empty, it is possible to consume and impossible to request.

where $AX^c \varphi := AG^{\{c\}} \varphi$, $AG \varphi := AG^{\Sigma^*} \varphi$, and $EX^c \varphi := EF^{\{c\}} \varphi$.

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$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Setting up the Scenery

- ▶ Σ alphabet, $|\Sigma| \geq 2$.
- ▶ Palindromes $:= \{w \in \Sigma^* \mid w = w^R\} \in \text{CFL} \setminus \text{DCFL}$.
- ▶ Extended alphabet $\Sigma_{\$} := \Sigma \dot{\cup} \{\$\}$.
- ▶ Goal: none $\text{PDL}_0[\text{DCFL}]$ -formula is equivalent to the $\text{PDL}_0[\text{CFL}]$ -formula $\text{EF}^{\text{Palindromes}\cdot\$}\text{tt}$.
- ▶ Assumption for contradiction: $\text{PDL}_0[\text{DCFL}] \ni \vartheta \equiv \text{EF}^{\text{Palindromes}\cdot\$}\text{tt}$.
- ▶ Wlog. no propositions in ϑ .

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —A Simple World

A simple world: \mathcal{V} uses neither \wedge nor **AG**.

- ▶ $\text{EF}^L \bigvee_i \psi_i \equiv \bigvee_i \text{EF}^L \psi_i$,
- ▶ $\text{EF}^{L_1} \text{EF}^{L_2} \psi \equiv \text{EF}^{L_1 \cdot L_2} \psi$, and
- ▶ $\text{EF}^L \text{ff} \equiv \text{ff}$.

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- ▶ $\text{EF}^L \bigvee_i \psi_i \equiv \bigvee_i \text{EF}^L \psi_i$,
- ▶ $\text{EF}^{L_1} \text{EF}^{L_2} \psi \equiv \text{EF}^{L_1 \cdot L_2} \psi$, and
- ▶ $\text{EF}^L \text{ff} \equiv \text{ff}$.

Therefore,

- ▶ $\vartheta \equiv \text{EF}^L \text{tt}$ where $L := \bigcup_i \prod_j L_{i,j}$ and $L_{i,j}$ are DCFLs over $\Sigma_{\$}$.
- ▶ $\text{Palindromes} = \{w \mid w\$ \text{ is a prefix of a word in } L\}$
 $= L / (\$ \cdot \Sigma_{\$}^*) \cap \Sigma^*$
as $\text{EF}^{\text{Palindromes} \cdot \$} \text{tt} \equiv \vartheta \equiv \text{EF}^L \text{tt}$.

In this world, **Palindromes** is a finite union of DCFL-concatenations.

$PDL_0[DCFL] \not\leq PDL_0[CFL]$ —The Real World

Theorem [Bojańczyk 2008]

For $L \subseteq \Sigma^\omega$ the following are equivalent:

- ▶ The tree language “a path belongs to L ” is definable in ECTL.
- ▶ L is a finite union of languages of the form

$$A_0^* a_1 A_1^* a_2 \cdots A_{n-1}^* a_n A_n^\omega$$

where $a_1, \dots, a_n \in \Sigma$ and $A_0, \dots, A_n \subseteq \Sigma$.

Intractability

\wedge - and AG -subformulas cannot be eliminated if the elimination method also applies to CTL.

$PDL_0[DCFL] \not\leq PDL_0[CFL]$ —Strategy

Planned Definition

A language L is **good** iff ...

Planned Theorem

If $EF^{L.\$} \text{tt} \equiv \vartheta \in PDL_0[DCFL]$ then L is good.

Planned Theorem

The language **Palindromes** is not good.

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$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Eliminations I

Theorem

If

$$\vartheta \equiv \text{EF}^L \text{tt}$$

then

$$\vartheta \equiv \bigvee_{\psi: \text{EF-formula in } \vartheta, \models \psi \rightarrow \text{EF}^L \text{tt}} \psi$$

$PDL_0[DCFL] \not\leq PDL_0[CFL]$ —Eliminations II

Proofsketch.

Transformations:

- ▶ Turn ϑ into a disjunctive normal form.
- ▶ Complete formula to

$$\vartheta \vee \bigvee \left\{ \bigwedge \Phi \mid \Phi \text{ set of EF-formulas in } \vartheta, \models \bigwedge \Phi \rightarrow \vartheta \right\}$$

Proofs:

- ▶ It is admissible to remove terms containing **AG**-formulas.
- ▶ In each term it suffices to pick one **EF**-formula. □

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Extraction I

Therefore, have for some DCFLs L_i over Σ .

$$\text{EF}^{\text{Palindromes} \cdot \$} \text{tt} \equiv \vartheta \equiv \bigvee_i \text{EF}^{L_i} \psi_i$$

Definiton

$$\llbracket \psi \rrbracket := \left\{ a_1 \cdots a_n \in \Sigma^* \mid \begin{array}{c} \rightarrow \bullet \xrightarrow{a_1} \bullet \cdots \cdots \xrightarrow{a_n} \bullet \xrightarrow{\$} \bullet \\ \vDash \psi \end{array} \right\}$$

$$\begin{aligned} \text{Thus, Palindromes} &= \bigcup_i (L_i \cap \Sigma^*) \cdot \llbracket \psi_i \rrbracket \cup (L_i / (\$ \Sigma^*) \cap \Sigma^*) \\ &= \bigcup_i L'_i \cdot R'_i \end{aligned}$$

for DCFLs L'_i over Σ , and for arbitrary languages R'_i over Σ .

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$\text{PDL}_0[\text{DCFL}] \not\subseteq \text{PDL}_0[\text{CFL}]$ —Palindromes vs. Concat.

Lemma

Suppose $LR \subseteq \text{Palindromes}$ and $|L| \geq 2$. Then

$$R \subseteq u^*V$$

for some word u and a **finite** language V .

$\text{PDL}_0[\text{DCFL}] \not\subseteq \text{PDL}_0[\text{CFL}]$ —Palindromes vs. Concat.

Proofsketch.

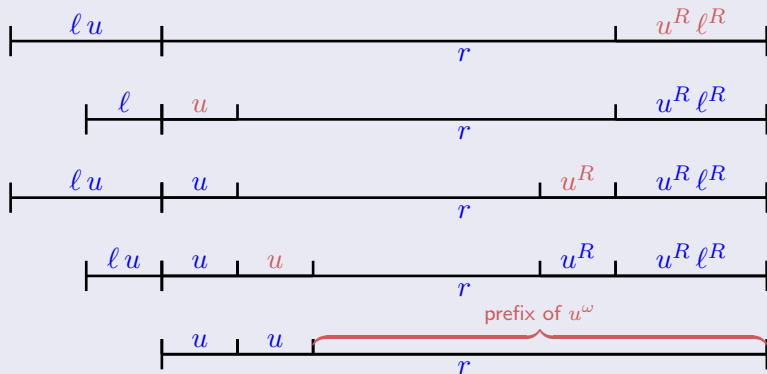
Let $l, lu \in L$ and $r \in R$.



$\text{PDL}_0[\text{DCFL}] \not\subseteq \text{PDL}_0[\text{CFL}]$ —Palindromes vs. Concat.

Proofsketch.

Let $l, lu \in L$ and $r \in R$.



$\text{PDL}_0[\text{DCFL}] \not\subseteq \text{PDL}_0[\text{CFL}]$ —Good vs. Bad

What can we say about R if $LR \subseteq \text{Palindromes}$ for $|L| = 1$?

$\text{PDL}_0[\text{DCFL}] \not\subseteq \text{PDL}_0[\text{CFL}]$ —Good vs. Bad

What can we say about R if $LR \subseteq \text{Palindromes}$ for $|L| = 1$?
Almost nothing: R could be $\text{Palindromes} \cdot L^R$.

Definition

A language L is **good** iff $L = \bigcup_{i=1}^n L_i R_i$ for L_i DCFL and $|L_i| \geq 2$.

Planned Theorem

If $\text{EF}^{L \cdot \$} \text{tt} \equiv \varnothing \in \text{PDL}_0[\text{DCFL}]$ then L is good.

Planned Theorem

The language **Palindromes** is not good.

$PDL_0[DCFL] \not\leq PDL_0[CFL]$ — Palindromes are Bad

Theorem

Palindromes is not good.

Proof.

$$\begin{aligned} \text{Otherwise, Palindromes} &= \bigcup_i L_i R_i \\ &= \bigcup_{\substack{i \\ R_i \text{ finite}}} \underbrace{L_i R_i}_{\text{DCFL}} \cup \bigcup_i u_{i,0} u_{i,1}^* u_{i,2}^* u_{i,3} \end{aligned}$$

Choose u s.t. it is not a prefix of the right union.

$$u \setminus \text{Palindromes} = \bigcup_i \underbrace{u \setminus L_i R_i}_{\text{DCFL}}$$

But Palindromes is not a finite union of DCFLs.



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Reminder

$\text{EF}^{\text{Palindromes}} \text{tt} \equiv \vartheta \equiv \bigvee_i \text{EF}^{L_i} \psi_i$ implies that

$$\text{Palindromes} = \bigcup_i L_i \cdot R_i$$

for DCFLs L_i over Σ , and for arbitrary languages R_i over Σ .

That's almost good except for “ $|L_i| > 1$ ”.

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Extraction III

Solution if $|L_i| = 1$ for some i

- ▶ Let $a \in \Sigma$ s.th. a is a prefix of the sole word of L_i .
- ▶ We have that $\text{EF}^{a \setminus \text{Palindromes}} \text{tt} \equiv \bigvee_i \text{EF}^{a \setminus L_i} \psi_i =: a \setminus \vartheta$.
- ▶ $\text{Palindromes} = \bigcup_i L_i R_i = \bigcup_{|L_i| \geq 2} L_i R_i \cup a \bigcup_i \underbrace{a \setminus L_i R_i}_{=[a \setminus \vartheta]}$

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Extraction III

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- ▶ $\text{Palindromes} = \bigcup_i L_i R_i = \bigcup_{|L_i| \geq 2} L_i R_i \cup a \bigcup_i \underbrace{a \setminus L_i R_i}_{=[a \setminus \vartheta]}$
- ▶ If $\llbracket a \setminus \vartheta \rrbracket$ is good then so **Palindromes**.

\rightsquigarrow Ind. on a suitable measure yields that **Palindromes** is good.

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Extraction IV

Definiton (Measure)

A measure is finite subset of $(\omega + 1)^*$, ordered by multi-set order based on lexicographical order on $(\omega + 1)^*$.

$$\mu(\ell) := \{[]\} \quad \ell \text{ literal or constant}$$

$$\mu(\varphi_1 \circ \varphi_2) := \mu(\varphi_1) \cup \mu(\varphi_2) \quad \circ \in \{\wedge, \vee\}$$

$$\mu(Q^L \varphi) := \begin{cases} \{|w| :: m \mid m \in \mu(\varphi)\} & \text{if } L = \{w\} \\ \{\omega :: m \mid m \in \mu(\varphi)\} & \text{otherwise} \end{cases} \quad Q \in \{\text{AG}, \text{EF}\}$$

Example: $\mu(\text{EF}^{\{abba\}}(\text{EF}^{\{c^n \mid n \leq 7\}} p \wedge \text{EF}^{\{ba\}} \neg p)) = \{[4, \omega], [4, 2]\}$

Properties

- ▶ All considered transformations weakly decrease the measure.
- ▶ $\mu(\text{EF}^L \psi) < \mu(\text{EF}^{a \setminus L} \psi)$ if $L = \{w\}$ and a prefix of w .

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$ —Extraction V

Theorem

If $\text{EF}^{L,\$} \text{tt} \equiv \vartheta \in \text{PDL}_0[\text{DCFL}]$ then L is good.

Corollary

$\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$.

Conclusion

- ▶ A separation of $\text{PDL}_0[\text{DCFL}]$ and $\text{PDL}_0[\text{CFL}]$ seems to be impossible by means of automata theory.
- ▶ Elimination of outermost \wedge - and AG -formulas is possible.
- ▶ Iterated elimination is non-uniform
—compared to Bojańczyk's impossibility result.
- ▶ $\text{PDL}_0[\text{DCFL}] \not\leq \text{PDL}_0[\text{CFL}]$.

Work in Progress and Future Work

- ▶ Extension to the **EU/AR**-fragment of $\text{XCTL}[\cdot] \geq \text{PDL}_0[\cdot]$.
Interpretation of $\mathbf{E}(\psi_1 \mathbf{U}^L \psi_2)$ compared to $\mathbf{EF}^L \psi$ as $L \cdot \llbracket \Psi \rrbracket$?
- ▶ Extension to the **PDL**[\cdot].
- ▶ Extension to the **EG/AF**-fragment and so to the whole $\text{XCTL}[\cdot]$.
- ▶ Extension to the $\Delta\text{PDL}^?[\cdot]$.
- ▶ Generalize to separations like:
 $\text{PDL}[\mathfrak{A}] \not\leq \text{PDL}[\mathfrak{B}]$ if \mathfrak{A} is a “reasonable” subset of \mathfrak{B} .

now $\models \text{EF}(\text{question answer})^* \text{AX ff}$