

# Implementation of Resource Aware JAva (RAJA)

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# Introduction

- System RAJA (Hofmann & Jost, ESOP 2006)
  - Type system for Java-like programs.
  - Compile-time analysis of heap-space requirements.
  - Amortised complexity analysis.
- Implementation in Ocaml
  - Result: Typechecker and interpreter for RAJA programs.
  - Challenge: Algorithmic presentation of the typing rules.
  - Goal: Testing and improving the system.

# Amortised Analysis of Heap-Usage

- Free-list based model
  - Deallocation in C/C++ style with primitive dispose.
  - Object creation: takes memory units from free-list.
  - Object destruction: returns memory units to free-list.
- Goal:
  - Upper bound on the size of the free-list.
  - As function of the input.
- Amortised analysis
  - Types assign each heap configuration statically a potential.
  - Object creation: must pay using potential from input.
  - Potential of consumed input: upper bound on heap space consumption.

# Object-Oriented Language: FJEU

$c ::=$	class $C$ [extends $D$ ] { $A_1; \dots; A_k; M_1 \dots M_j$ }
$A ::=$	$C$ $a$
$M ::=$	$C_0$ $m(C_1$ $x_1, \dots, C_j$ $x_j)$ {return $e$ ; }
$e ::=$	$x$ (Variable)
	null (Constant)
	new $C$ (Construction)
	free ( $x$ ) (Destruction)
	( $C$ ) $x$ (Cast)
	$x.a_i$ (Access)
	$x.a_i \leftarrow x$ (Update)
	$x.m(x_1, \dots, x_j)$ (Invocation)
	if $x$ instanceof $C$ then $e_1$ else $e_2$ (Conditional)
	let $x = e_1$ in $e_2$ (Let)

- $\approx$  Featherweight Java + imperative field update.

# Copy example

In Java

```
abstract class List {
    abstract List copy();
}

class Nil extends List {
    List copy() {
        return this;
    }
}

class Cons extends List {
    int elem;
    List next;

    List copy() {
        Cons res = new Cons;
        res.elem = this.elem;
        res.next = this.next.copy();
        return res;
    }
}
```

In FJEU

```
class List {
    List copy(){ return null; }
}

class Nil extends List {
    List copy() {
        return this;
    }
}

class Cons extends List {
    int elem;
    List next;

    List copy() {
        let Cons res = new Cons in
        let int elem = this.elem in
        let Cons res1 = res.elem ← elem in
        let List next = this.next in
        let List rnext = next.copy() in
        let List res2 = res1.next ← rnext in
        return res2;
    }
}
```

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        let List rnext = next.copy() in
        let List res2 = res1.next ← rnext in
        return res2;
    }
}
```

# Sketch of RAJA System

- RAJA program  $P = \textit{annotated}$  FJEU program.
  - 1 Set of *views*  $\mathcal{V}$ .
  - 2 For each class  $C$  and view  $r$  we have an annotated version  $C^r$ .
  - 3 Potential:
    - $\diamond(C^r) : \text{Class} \times \text{View} \rightarrow \mathbb{Q}^+$
  - 4 (Get- and set-) views for attributes:
    - $A^{\text{get}}(C^r, a) : \text{Class} \times \text{View} \times \text{Field} \rightarrow \text{View}$  (get-view)
    - $A^{\text{set}}(C^r, a) : \text{Class} \times \text{View} \times \text{Field} \rightarrow \text{View}$  (set-view)
  - 5 RAJA types for methods:
    - $M(C^r, m) : \text{Class} \times \text{View} \times \text{Method} \rightarrow$   
 $\mathcal{P}(\text{Views of Arguments} \rightarrow \text{Effect} \times \text{View of Result})$   
 $(r_1, \dots, r_j \xrightarrow{m/m'} r_0)$
    - Effect: pair of numbers  $m, m'$  representing potential consumed and released by the method.

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 $(r_1, \dots, r_j \xrightarrow{m/m'} r_0)$
    - Effect: pair of numbers  $m, m'$  representing potential consumed and released by the method.

# Copy example in RAJA

```

views rich, poor
class List implements rich, poor {
  rich:pot = 0;
  poor:pot = 0;
  rich>List<poor>,0 copy() { return null;}
}
class Nil extends List implements rich, poor {
  rich:pot = 0;
  poor:pot = 0;
  rich>List<poor>,0 copy() { return this; }
}
class Cons extends List implements rich, poor {
  rich:pot = 1;
  poor:pot = 0;
  rich:int elem;
  poor:int elem;
  rich>List<rich,rich> next;
  poor>List<poor,poor> next;

  rich>List<poor>,0 copy() {
    Cons<poor> res = new Cons;
    res.elem = elem;
    res.next = this.next.copy();
    return res;
  }
}

```

$\diamond(\cdot)$	rich	poor
List	0	0
Nil	0	0
Cons	1	0

	Cons <sup>rich</sup>	Cons <sup>poor</sup>
A <sup>get</sup> ( $\cdot$ , next)	rich	poor
A <sup>set</sup> ( $\cdot$ , next)	rich	poor

	List <sup>rich</sup>	Nil <sup>rich</sup>	Cons <sup>rich</sup>
M( $\cdot$ , copy)	() $\xrightarrow{0/0}$ poor		

Meaning of *rich* and *poor*:

potential( $l : \text{List}^{\text{rich}}$ ) = length( $l$ )  
 potential( $l : \text{List}^{\text{poor}}$ ) = 0

we pay the copy of  $l : \text{List}^{\text{rich}}$  from its potential but we *cannot* copy  $l : \text{List}^{\text{poor}}$

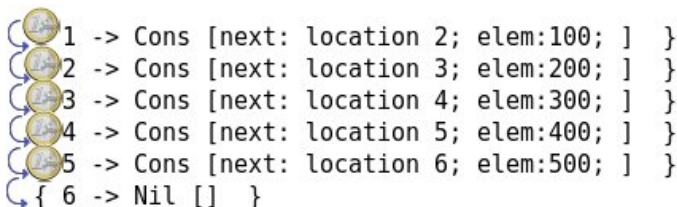
## How is potential defined?

- Object *cons* : *List<sup>rich</sup>* in the heap pointing to location 1
- [100, 200, 300, 400, 500]
- $\text{length}(\text{cons}) = 5$

```
{ 1 -> Cons [next: location 2; elem:100; ] }
{ 2 -> Cons [next: location 3; elem:200; ] }
{ 3 -> Cons [next: location 4; elem:300; ] }
{ 4 -> Cons [next: location 5; elem:400; ] }
{ 5 -> Cons [next: location 6; elem:500; ] }
{ 6 -> Nil [] }
```

Figure: Heap configuration

## How is potential defined?



$$\phi_{\sigma}((\text{cons} : \text{rich}).\epsilon) = \diamond(\text{Cons}^{\text{rich}}) = 1$$

$$\phi_{\sigma}((\text{cons} : \text{rich}).\text{next}) = \diamond(\text{Cons}^{\text{Aget}(\text{Cons}^{\text{rich}}, \text{next})}) = 1 \quad \dots$$

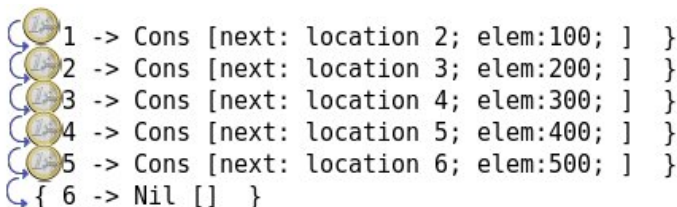
$$\phi_{\sigma}((\text{cons} : \text{rich}).\text{next}.\text{next}.\text{next}.\text{next}) = 1$$

$$\phi_{\sigma}((\text{cons} : \text{rich}).\text{next}.\text{next}.\text{next}.\text{next}.\text{next}) = \diamond(\text{Nil}^{\text{rich}}) = 0$$

$$\text{Therefore: } \Phi_{\sigma}(\text{cons} : \text{rich}) = \sum_{\vec{p}} \phi_{\sigma}((\text{cons} : \text{rich}).\vec{p}) = 5$$

$$\text{but } \Phi_{\sigma}(\text{cons} : \text{poor}) = 0$$

## How is potential defined?



- $$\phi_{\sigma}((cons: rich).\epsilon) = \diamond(\text{Cons}^{rich}) = 1$$

$$\phi_{\sigma}((cons: rich).next) = \diamond(\text{Cons}^{A_{\text{get}}(\text{Cons}^{rich}, \text{next})}) = 1 \quad \dots$$

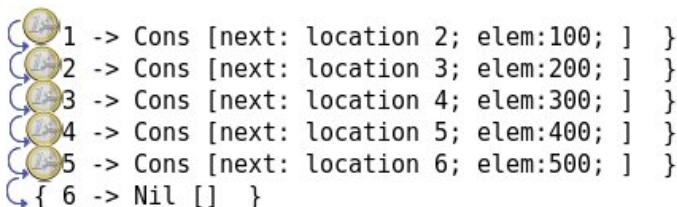
$$\phi_{\sigma}((cons: rich).next.next.next.next) = 1$$

$$\phi_{\sigma}((cons: rich).next.next.next.next.next) = \diamond(\text{Nil}^{rich}) = 0$$

Therefore:  $\Phi_{\sigma}(cons : rich) = \sum_{\vec{p}} \phi_{\sigma}((cons: rich).\vec{p}) = 5$

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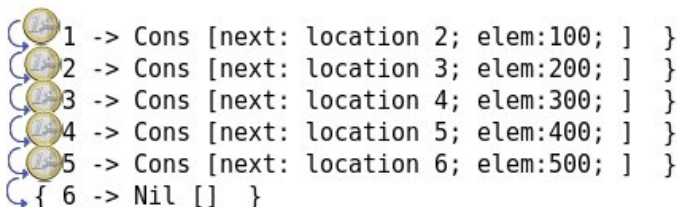
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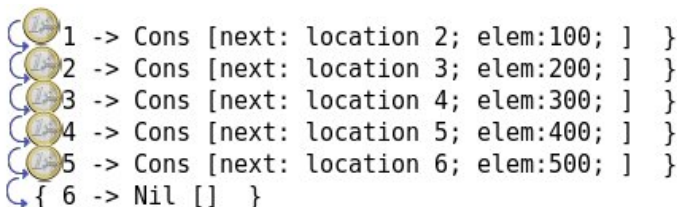
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Therefore:  $\Phi_\sigma(\text{cons} : \text{rich}) = \sum_{\vec{p}} \phi_\sigma((\text{cons} : \text{rich}).\vec{p}) = 5$

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- $$\phi_\sigma((cons : rich).\epsilon) = \diamond(\text{Cons}^{rich}) = 1$$

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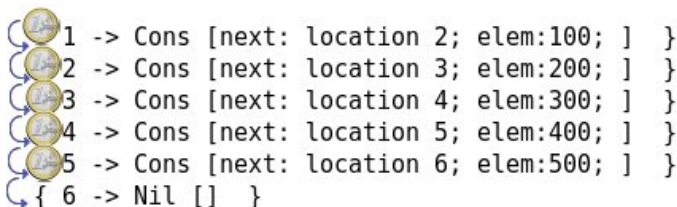
$$\phi_\sigma((cons : rich).next.next.next.next) = 1$$

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Therefore:  $\Phi_\sigma(cons : rich) = \sum_{\vec{p}} \phi_\sigma((cons : rich).\vec{p}) = 5$

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$$\text{Therefore: } \Phi_{\sigma}(cons : rich) = \sum_{\vec{p}} \phi_{\sigma}((cons : rich).\vec{p}) = 5$$

$$\text{but } \Phi_{\sigma}(cons : poor) = 0$$

# Typing and semantics

- Semantics:  $\eta, \sigma \stackrel{n}{n'} \circ e \rightsquigarrow v, \tau$  means:
  - expression  $e$  evaluates successfully to value  $v$  beginning with stack  $\eta$  and heap  $\sigma$  and ending with heap  $\tau$ .
  - $n$ : unused heap units before evaluating  $e$ .
  - $n'$ : unused heap units after evaluating  $e$ .
- Typing judgement  $\Gamma \stackrel{n}{n'} e : C^r$ .

## Theorem

If  $\Gamma \stackrel{n}{n'} e : C^r$  and  $\eta, \sigma \stackrel{\circ}{\circ} e \rightsquigarrow v, \tau$  then

$$\eta, \sigma \stackrel{n + \Phi_{\sigma}(\eta : \Gamma) + \Phi_{\sigma}(\eta : \Delta)}{n' + \Phi_{\tau}(v : r) + \Phi_{\tau}(\eta : \Delta)} \circ e \rightsquigarrow v, \tau$$

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# Typing and semantics: new and free

$$\frac{}{\emptyset \vdash \frac{n}{n - (\diamond(C^r) + \text{Size}(C))} \text{ new } C : C^r} (\diamond \text{New})$$

$$\frac{\tau = \sigma[v \mapsto (C, a_1:0, \dots, a_k:0)]}{\eta, \sigma \vdash \frac{n + \text{Size}(C)}{n} \circ \text{ new } C \rightsquigarrow v, \tau} (\vdash \text{New})$$

$$\frac{}{x : C^r \vdash \frac{n}{n + \diamond(C^r) + \text{Size}(C)} \text{ free } (x) : E^r} (\diamond \text{Free})$$

$$\frac{\eta_x = v \quad \sigma_v = (C, a_1:v_1, \dots, a_k:v_k)}{\eta, \sigma \vdash \frac{n}{n + \text{Size}(C)} \circ \text{ free } (x) \rightsquigarrow 0, (\sigma[v \mapsto \text{invalid}])} (\vdash \text{Free})$$

# RAJA typing rules

$$\frac{A^{\text{get}}(C^r, a) = s \quad C.a = D}{x : C^r \vdash_n^{\frac{n}{n}} x.a : D^s} \quad (\diamond \text{Access})$$

$$\frac{A^{\text{set}}(C^r, a) = s \quad C.a = D}{x : C^r, y : D^s \vdash_n^{\frac{n}{n}} x.a \leftarrow y : C^r} \quad (\diamond \text{Update})$$

$$\frac{(E_1^{q_1}, \dots, E_j^{q_j} \xrightarrow{m/m'} E_0^{q_0}) \in M(C^r, m) \quad n \geq m}{x : C^r, y_1 : E_1^{q_1}, \dots, y_j : E_j^{q_j} \vdash_{m' + n - m}^{\frac{n}{n}} x.m(y_1, \dots, y_j) : E_0^{q_0}} \quad (\diamond \text{Invocation})$$

$$\frac{x \in \Gamma \quad \Gamma \vdash_{n'}^{\frac{n}{n'}} e_1 : C^r \quad \Gamma \vdash_{n''}^{\frac{n}{n''}} e_2 : C^r}{\Gamma \vdash_{\max(n', n'')}^{\frac{n}{n}} \text{if } x \text{ instance of } E \text{ then } e_1 \text{ else } e_2 : C^r} \quad (\diamond \text{Conditional})$$

# Sharing relation

- $\forall(r | s_1, \dots, s_j)$ : coinductively defined relation between view  $r$  and the multiset of views  $s_1, \dots, s_j$ :

for all  $C : \diamond(C^r) \geq \diamond(C^{s_1}) + \dots + \diamond(C^{s_j})$ , etc.

$\diamond(\cdot)$	rich	poor	rest	first	poorest
List	0	0	0	0	0
Nil	0	0	0	0	0
Cons	1	0	0	1	0

	Cons <sup>rich</sup>	Cons <sup>poor</sup>	Cons <sup>rest</sup>	Cons <sup>first</sup>	Cons <sup>poorest</sup>
A <sup>get</sup> ( $\cdot$ , next)	rich	poor	rich	poorest	poorest
A <sup>set</sup> ( $\cdot$ , next)	rich	poor	rich	rich	rich

- $\forall(\text{rich} | \text{rich}, \text{poorest})$
- $\forall(\text{rich} | \text{rest}, \text{first})$
- NOT  $\forall(\text{rich} | \text{rich}, \text{rich})$

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A <sup>get</sup> ( $\cdot$ , next)	rich	poor	rich	poorest	poorest
A <sup>set</sup> ( $\cdot$ , next)	rich	poor	rich	rich	rich

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- $\forall(\text{rich} | \text{rest}, \text{first})$
- NOT  $\forall(\text{rich} | \text{rich}, \text{rich})$

## Sharing relation: examples

- $cons : List^{rich}$

```
1 -> Cons [next: location 2; elem:100; ]  
2 -> Cons [next: location 3; elem:200; ]  
3 -> Cons [next: location 4; elem:300; ]  
4 -> Cons [next: location 5; elem:400; ]  
5 -> Cons [next: location 6; elem:500; ]  
{ 6 -> Nil [] }
```

- $cons : List^{poorest}$

```
{ 1 -> Cons [next: location 2; elem:100; ]  
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```

- $\Downarrow(rich | rich, poorest)$

## Sharing relation: examples

- $cons : List^{first}$

```
1 -> Cons [next: location 2; elem:100; ]  
{ 2 -> Cons [next: location 3; elem:200; ]  
{ 3 -> Cons [next: location 4; elem:300; ]  
{ 4 -> Cons [next: location 5; elem:400; ]  
{ 5 -> Cons [next: location 6; elem:500; ]  
{ 6 -> Nil [] }
```

- $cons : List^{rest}$

```
{ 1 -> Cons [next: location 2; elem:100; ]  
2 -> Cons [next: location 3; elem:200; ]  
3 -> Cons [next: location 4; elem:300; ]  
4 -> Cons [next: location 5; elem:400; ]  
5 -> Cons [next: location 6; elem:500; ]  
{ 6 -> Nil [] }
```

- $\forall (rich | rest, first)$

## Gaining potential from *this*

$$\frac{\forall(r|q,s) \quad M(C^r, m) \ni E_1^{r_1}, \dots, E_j^{r_j} \xrightarrow{n/n'} E_0^{r_0}}{\text{this}: C^q, x_1: E_1^{r_1}, \dots, x_j: E_j^{r_j} \mid \frac{n + \diamond(C^s)}{n'} \quad M_{\text{body}}(C, m) : E_0^{r_0}}$$

- Implementation without inference:

```
rich as <rest, first>:List<poor>, 0 copy(0) {  
    Cons<poor> res = new Cons;  
    res.elem = elem;  
    res.next = this.next.copy();  
    return res;  
}
```

- Typechecker checks sharing:  $\forall(\text{rich} | \text{rest}, \text{first})$

## Gaining potential from this

$$\frac{\forall(r|q,s) \quad M(C^r, m) \ni E_1^{r_1}, \dots, E_j^{r_j} \xrightarrow{n/n'} E_0^{r_0}}{\text{this}: C^q, x_1: E_1^{r_1}, \dots, x_j: E_j^{r_j} \mid \frac{n + \diamond(C^s)}{n'} \quad M_{\text{body}}(C, m) : E_0^{r_0}}$$

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- Typechecker checks sharing:  $\forall(\text{rich} \mid \text{rest}, \text{first})$

## Share rule

- Handling of aliasing: sharing rule

$$\frac{\forall(s \mid q_1, q_2) \quad \Gamma, y:D^{q_1}, z:D^{q_2} \vdash_{\frac{n}{n'}} e : C^r}{\Gamma, x:D^s \vdash_{\frac{n}{n'}} e[x/y, x/z] : C^r} (\diamond Share)$$

# Implementation of sharing

- Problems with sharing:
  - ① ( $\diamond$ Share) not syntax directed.
  - ② Sharing needs to be guessed or calculated.
- Implementation without inference: annotation of every variable occurrence.

$$\frac{}{\Gamma, x: C(r|q_1, \dots, q_i), y: D(s|q'_1, \dots, q'_j) \vdash_n [x \text{ as } q]. a \leftarrow [y \text{ as } q'] : C^q} \quad (\diamond \text{Update})$$

- every rule updates the context:

$$\Gamma, x: C(r|q_1, \dots, q_i, q), y: D(s|q'_1, \dots, q'_j, q')$$

- after typechecking, sharing is checked:

$$\text{for every } x: C(r|q_1, \dots, q_n) \in \Gamma : \text{check } \forall (r | q_1, \dots, q_n)$$

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# Implementation and Examples

- More implementation features:
  - Typechecker for RAJA programs: prediction of free-list size.
  - Interpreter for FJEU programs with built-in calculation of free-list size.
  - Integers as primitives.
  - Arithmetic expressions.
  - Booleans as built-in classes Bool, True and False.
- Examples
  - ① Copy of lists: correct prediction of linear space consumption.
  - ② Handling with circular data: doubly linked lists.
  - ③ More computations: merge sort.
- Demo...

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  - Typechecker for RAJA programs: prediction of free-list size.
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  - 1 Copy of lists: correct prediction of linear space consumption.
  - 2 Handling with circular data: doubly linked lists.
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# Conclusions and further work

- Conclusions
  - Our type-based analysis encompasses:
    - ① Objects
    - ② Inheritance
    - ③ Downcast
    - ④ Imperative update
    - ⑤ Aliasing
    - ⑥ Circular data
  - Implementation allows:
    - Experimenting with the system
    - Testing the system with bigger programs
- Further work
  - More examples: Iterators on lists, etc.
  - Improving the type system with new features: polymorphism, etc.
  - Type inference