Optimization of a verified Deflate implementation

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An informal illustration of the format:

\[
\text{Deflate} ::= (\text{'}0\text{'} \text{ Block})^* \text{'}1\text{'} \text{ Block} (0|1)^*
\]

\[
\text{Block} ::= \text{'}00\text{'} \text{ UncompressedBlock} | \text{'}01\text{'} \text{ DynamicallyCompBl} | \text{'}10\text{'} \text{ StaticallyCompBl}
\]

\[
\text{UncompressedBlock} ::= \text{length} \sim \text{length bytes}
\]

\[
\text{StaticallyCompBl} ::= \text{CompBl(standard coding)}
\]

\[
\text{DynamicallyCompBl} ::= \text{header coding CompBl(coding)}
\]

\[
\text{CompBl(c)} ::= [{^256}]* 256
\]

Does not require any specific compression algorithm, even though some are recommended \(\Rightarrow\) we use encoding relations rather than directly writing a function.
What we have

- Mathematical specification of a compression format that is very likely Deflate
  - Tested it with lots of randomized data
  - And with real-life data
  - However, standards are axiomatic
- Implementation of a decompression algorithm through program extraction for testing
Pros and Cons of Extraction

Pros:
- Sophisticated formats should come with (at least informal) correctness proofs anyway

Cons:
- Proofs must be (mostly) constructive
- Harder to see the computational complexity, especially in presence of tactics - but: Semi-automatic theorem proving and program optimizers get better.

Coq can do both verification of functions as well as program extraction from proofs.
A little argument

“If the verified program is not better than compressing and decompressing with the unverified program, it is worthless”:

- Assume unverified algorithms $c, d$. Then the following are verified algorithms:

$$
\text{vc } c \quad d \quad x = \text{let } y = c \times \\
\quad \quad \text{in if } (d \quad y = x) \text{ then } (true, y) \\
\quad \quad \quad \text{else } (false, x)
$$

$$
\text{vd } d (b, x) = \text{if } b \text{ then } d \times x \text{ else } x
$$

- But this assumes that $c$ and $d$ are always the same on every platform.
- It does not allow later optimisations or even compatible bugfixes.
- $\Rightarrow$ better to have a formal specification and verify against it.
- Still a good “benchmark”.

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Problems

- Large trusted codebase:
  - Coq
  - GHC
  - Coq extraction mechanism
- Slow, probably superquadratic.
- Memory-Consuming.
Coq does not have intrinsics/builtins:

- Proving stuff about `nat` is easier and more usual than about `N` (binary integers).
- It is possible to extract inductive predicates to other datatypes:

  ```coq
  Extract Inductive nat ⇒
  "Prelude.Int" [ "0" "(1 Prelude.+)"]
  "( let r z s n = case n of {
      0 → z 0 ;
      q → s (q Prelude.- 1)}
      in r )".
  ```

- But Coq by default extracts all functions (even `+`, `*`) to their recursive definitions (thus, addition takes linear time, multiplication takes quadratic time, etc.).
Overloading

It is possible to overload defined functions:

Extract Constant plus ⇒ "(Prelude.+)".

Extract Constant mult ⇒ "(Prelude.*)".

However, this makes the trusted code base larger, and we cannot overload it differently on other parts of the program. It would be nicer to have some mechanism for this (maybe univalence will become something like that?)
Benchmarks suggested that there are lots of list append, snoc and nth-calls.

The advantage of lists is that they are a simple well-understood concept with a very small surface for bugs.

The disadvantage is that the said operations are slow.

⇒ We used Extract Constant and a catenable deque (by Chris Okasaki)
Catenable Deques - 2

Program $O(n \log n^2)$

* 512 Bytes

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Deflate in Coq
The runtime is still much too slow, but at least in a reasonable complexity class.

However, it uses tremendous amounts of memory:
- The catenable deque structure we used prevents Haskell from reading the file lazily, therefore the whole file must be loaded.
- The file is loaded bitwise into this structure, but Haskell does not optimize bit lists: A pointer and a box is stored for every bit of the file.
- The main culprit appears to be the resolution of backreferences.
A backreference is a pair $\langle l, d \rangle$ of a length $l$ (number of bytes to be copied) and a distance $d$ (number of recently extracted bytes that have to be skipped).

$\text{ananas\_banana\_batata} \Rightarrow \text{ananas\_b}\langle 5, 8 \rangle\langle 3, 7 \rangle\text{tata}$

An intuitive algorithm would be:

```haskell
resolve :: Int -> Int -> Int -> STArray s Int Int -> ST s Int
resolve len dist ptr out =
  if len > 0
    then do byte <- readArray out $ ptr - dist
            writeArray out ptr byte
            resolve (len - 1) dist (ptr + 1) out
    else do return ptr
```

$\Rightarrow$ Works also when $l > d$, results in a repetition of already written bytes $\Rightarrow$ Run-length-encoding
Factored out the resolution of backreferences, it is the last thing that is done.

Tried to use reverse lists (so `snoc`-operations are faster), but still the runtime was bad.

With the catenable deques, it performed better but consumed too much memory.

Current implementation uses “ExpList”:

```coq
Inductive ExpList (A : Set) : Set :=
| Enil : ExpList A
| Econsl : A → ExpList (A * A) → ExpList A
```
The advantage of ExpLists is that \texttt{nth} and \texttt{cons} consume logarithmic time.

The output list is combined with a cache

The cache contains two ExpLists

If the first ExpList fills up (has as much elements as our cache should have), it becomes the second ExpList, and the original second ExpList is dismissed, and a new empty first ExpList is allocated $\Rightarrow$ limits memory overhead.
The file `gzip-1.3.3.tar.gz` is 308K large. With backreference resolution the way we do it, it takes 46 minutes to decompress - without resolution, 15 seconds (on alicante - on siena 40 seconds).

That is, resolution makes the implementation a magnitude slower.

We are currently trying to use other techniques for optimizing it:

- DiffArrays - a simple hack that will probably work satisfactorily.
- Ynot - only extracts to OCaml.
- VST - the specifications are very complicated.