Uebungen am 15.11.2001

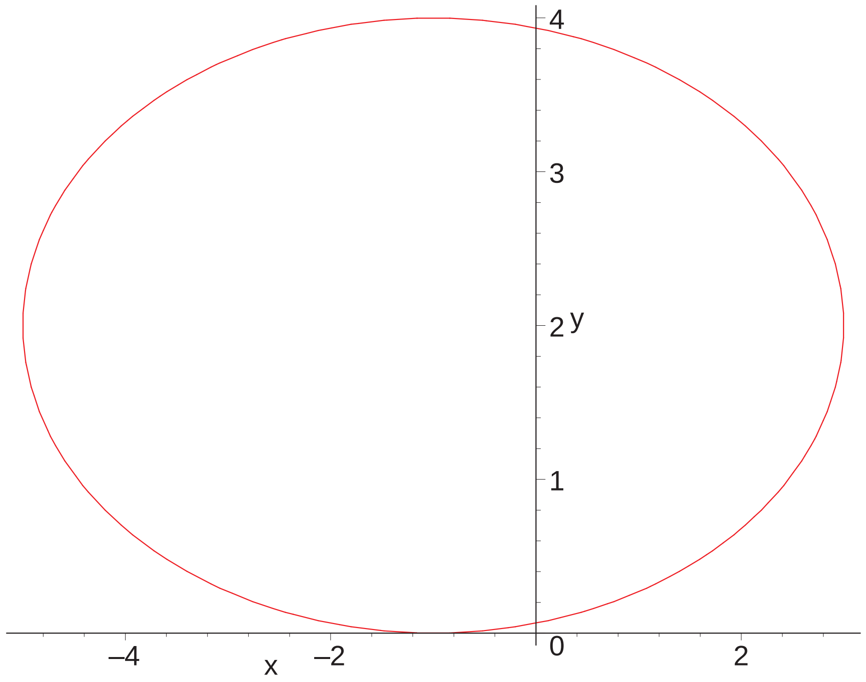
zu "Computergestuetztes symbolisches Rechnen" im Wintersemester 2001/02, Institut fuer Informatik der Universitaet Muenchen

Die Ellipse

```
> restart;
> ?implicitplot
> ellipse:=x^2+4*y^2+2*x-16*y+1=0;
```

$$ellipse := x^2 + 4\,y^2 + 2\,x - 16\,y + 1 = 0$$

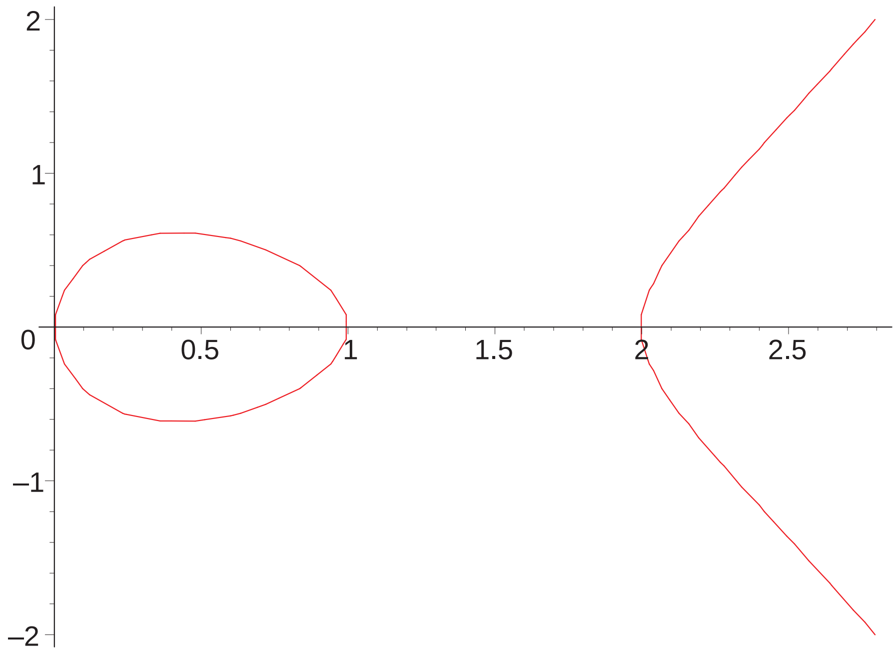
```
> with(plots):
Warning, the name changecoords has been redefined

> implicitplot(ellipse,x=-5..3,y=0..4);
```



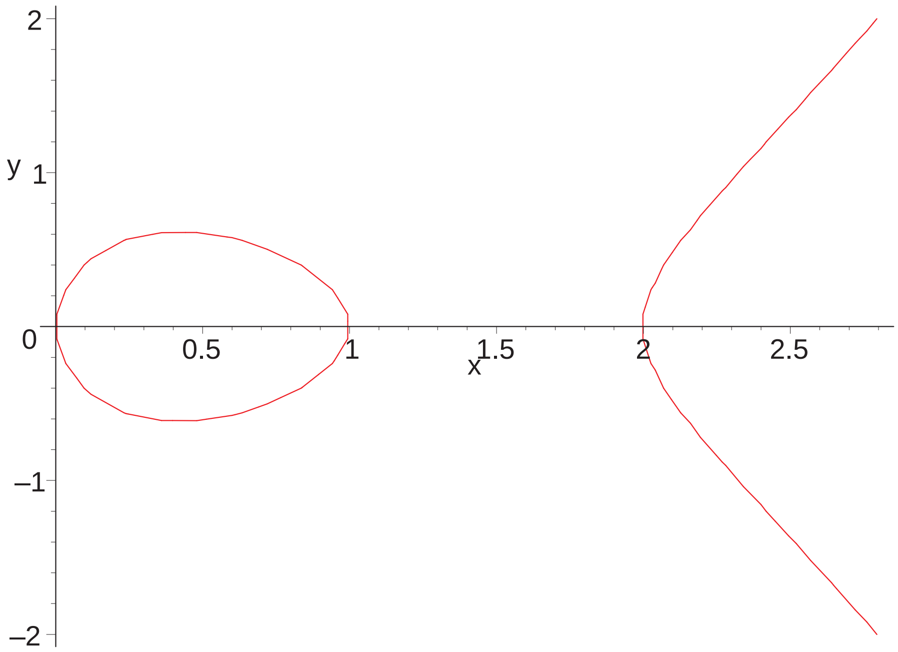

```
> funk2:=proc(x,y)y^2-x*(x-1)*(x-2)end proc;
```

$$funk2 := \textbf{proc}(x,\, y)\; y\text{^}2 - x*(x-1)*(x-2)\;\textbf{end proc}$$

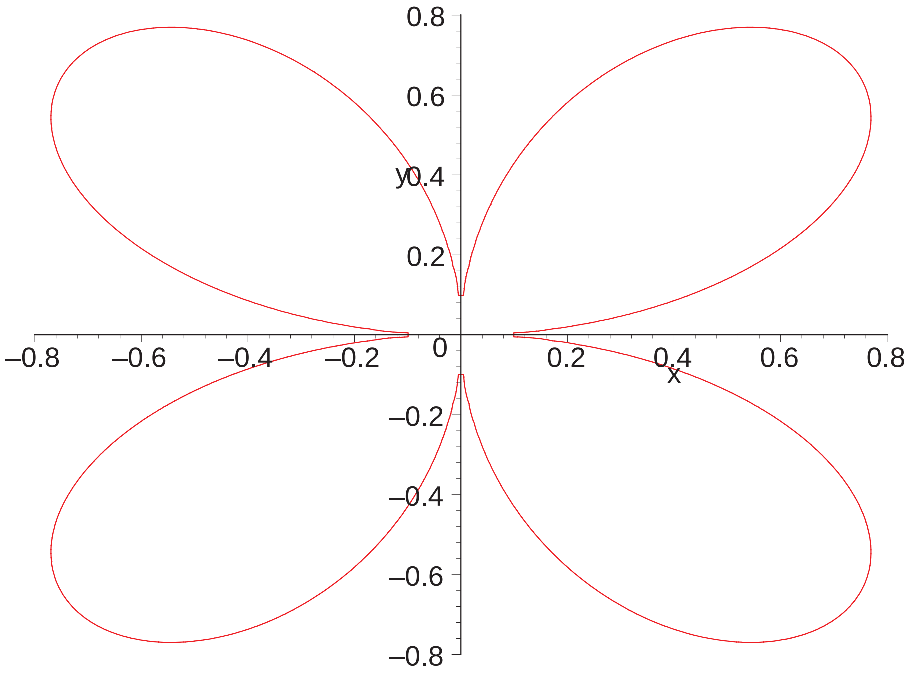
```
> implicitplot(funk2,0..3,-2..2);
```




```
> implicitplot(y^2-x*(x-1)*(x-2)=0,x=0..3,y=-2..2);
```




```
> implicitplot((x^2+y^2)^3-4*x^2*y^2,x=-1..1,y=-1..1,
  grid=[200,200]);
```

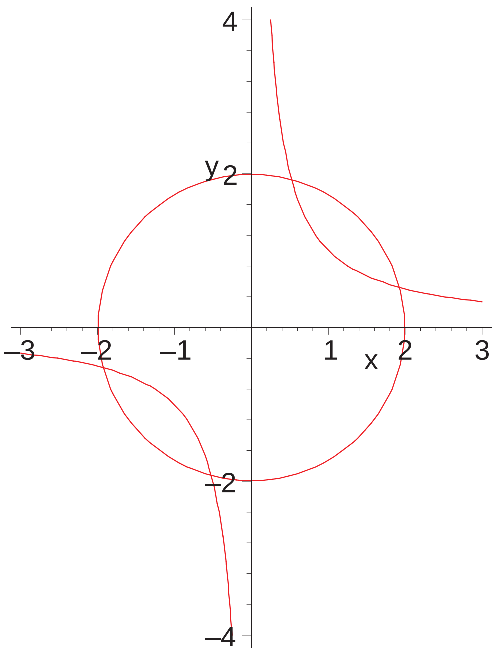



```
> ?solve
> solve((x^2+y^2)^3-4*x^2*y^2=0,x);
```

$$\frac{1}{3}\,\mathrm{sqrt}(-3\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}$$
$$(-(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12+3\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)})\,y)\;\Big/$$
$$(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)},\;-\frac{1}{3}\,\mathrm{sqrt}(-3\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}$$
$$(-(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12+3\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)})\,y)\;\Big/$$
$$(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)},\;\frac{1}{6}\sqrt{6}\,\mathrm{sqrt}((-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}\,($$
$$-(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12-6\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}$$
$$+I\sqrt{3}\,(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12\,I\sqrt{3})\,y)\;\Big/\;(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)},\;-\frac{1}{6}\sqrt{6}$$
$$\mathrm{sqrt}((-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}\,(-(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12$$
$$-6\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}+I\sqrt{3}\,(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12\,I\sqrt{3})\,y)\;\Big/$$
$$(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)},\;\frac{1}{6}\,\mathrm{sqrt}(-6\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}\,($$
$$(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}+12+6\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}$$

$$+I\sqrt{3}\,(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12\,I\sqrt{3}\,)\,y)\;\Big/\;(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)},\;-\frac{1}{6}\,\mathrm{sqrt}($$
$$-6\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}\,((-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}+12$$
$$+6\,y\,(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}+I\sqrt{3}\,(-54\,y+6\sqrt{-48+81\,y^2})^{(2/3)}-12\,I\sqrt{3}\,)\,y)\;\Big/$$
$$(-54\,y+6\sqrt{-48+81\,y^2})^{(1/3)}$$

```
> implicitplot({x^2+y^2-4=0,x*y=1},x=-3..3,y=-4..4,scaling=constra
  ined);
```




```
> solve({x^4-z*x=0,  x^3-y*x=0},{x,y});
```

$$\{x=0,\,y=y\},\,\{x=\mathrm{RootOf}(\_Z^3-z,\,label=\_L2),\,y=\mathrm{RootOf}(\_Z^3-z,\,label=\_L2)^2\}$$

```
> loes:=%[2];
```

$$loes:=\{x=\mathrm{RootOf}(\_Z^3-z,\,label=\_L2),\,y=\mathrm{RootOf}(\_Z^3-z,\,label=\_L2)^2\}$$

```
> allvalues(eval(x,loes));
```

$$z^{(1/3)},\,z^{(1/3)}\,(\text{-}1)^{(2/3)},\,-z^{(1/3)}\,(\text{-}1)^{(1/3)}$$

```
> groebnerbase({x^2-y^2-4,x*y},{x,y});
```

$$\mathrm{groebnerbase}(\{x\,y,\,x^2-y^2-4\},\,\{y,\,x\})$$

```
> ?groebne
> ?Groebner
> with(Groebner,gbasis);
```

$$[gbasis]$$

```
> Basis:=gbasis({x^2+y^2-4,x*y-1},plex(x,y));
```

$$Basis:=[1+y^4-4\,y^2,\,x+y^3-4\,y]$$

```
> Basis1:=gbasis({x^2+y^2-4,x*y-1},plex(y,x));
```

$$Basis1 := [1+x^4-4x^2,\ y+x^3-4x]$$

```
> with(Groebner);
```

$[fglm, gbasis, gsolve, hilbertdim, hilbertpoly, hilbertseries, inter\_reduce, is\_finite, is\_solvable, leadcoeff, leadmon, leadterm, normalf, pretend\_gbasis, reduce, spoly, termorder, testorder, univpoly]$

```
> testorder(x^2*y,x*y^2,plex(x,y));
```

$$false$$

```
> testorder(x^2*y,x*y^2,plex(y,x));
```

$$true$$

```
> normalf(x^2+y^2-4,Basis1,plex(x,y));
```

$$x^2+y^2-4$$

```
> normalf(x^2+y^2-4,Basis,plex(x,y));
```

$$0$$

```
> loesungen:=solve(Basis[1],y);
```

$$loesungen := -\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ -\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}$$

```
> expand(simplify(loesungen[3]^2));
```

$$2-\sqrt{3}$$

```
> ?eval
> eval(4*y-y^3,y=loesungen[3]);
```

$$-\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)^3-2\sqrt{6}+2\sqrt{2}$$

```
> simplify(%);
```

$$-\frac{1}{2}\sqrt{2}\sqrt{3}-\frac{1}{2}\sqrt{2}$$

```
> %+(2+sqrt(3))*sqrt(2-sqrt(3));
```

$$-\frac{1}{2}\sqrt{2}\sqrt{3}-\frac{1}{2}\sqrt{2}+(2+\sqrt{3})\left(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right)$$

```
> expand(%);
```

$$-\sqrt{2}\sqrt{3}-\frac{3}{2}\sqrt{2}+\sqrt{6}+\frac{1}{2}\sqrt{3}\sqrt{6}$$

```
> simplify(%);
```

$$0$$

```
> solve({x^2+y^2-4=0,x*y=1},{x,y});
```

$$\{y=\mathrm{RootOf}(1+\_Z^4-4\_Z^2),\ x=-\mathrm{RootOf}(1+\_Z^4-4\_Z^2)^3+4\,\mathrm{RootOf}(1+\_Z^4-4\_Z^2)\}$$

```
> allvalues(%);
```

$$\{y=-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ x=-\left(-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right)^3-2\sqrt{6}-2\sqrt{2}\},$$

$$\{y=\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ x=-\left(\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)^3+2\sqrt{6}+2\sqrt{2}\},$$

$$\{y=-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ x=-\left(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right)^3-2\sqrt{6}+2\sqrt{2}\},$$

$$\{y=\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ x=-\left(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right)^3+2\sqrt{6}-2\sqrt{2}\}$$

```
> ?polynomial
> type(x*y-5*x^3/y,polynom);
```

false

```
> type(x*y-5*x^3/x,polynom);
```

true

```
> type(sin(x)^2+cos(x)^2,polynom);
```

false

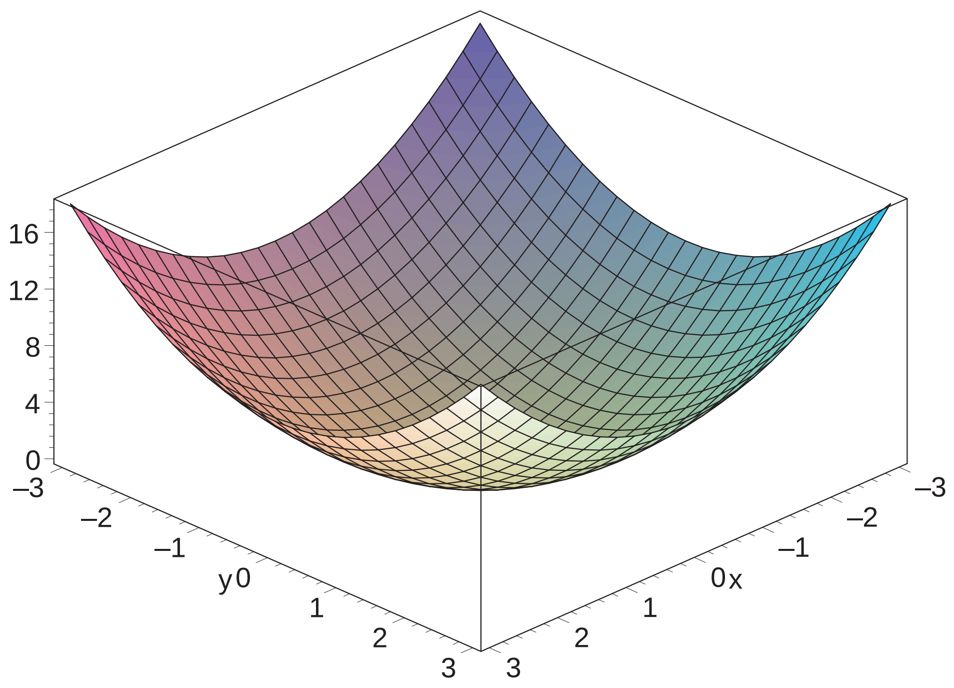
```
> type(simplify(sin(x)^2+cos(x)^2),polynom);
```

true

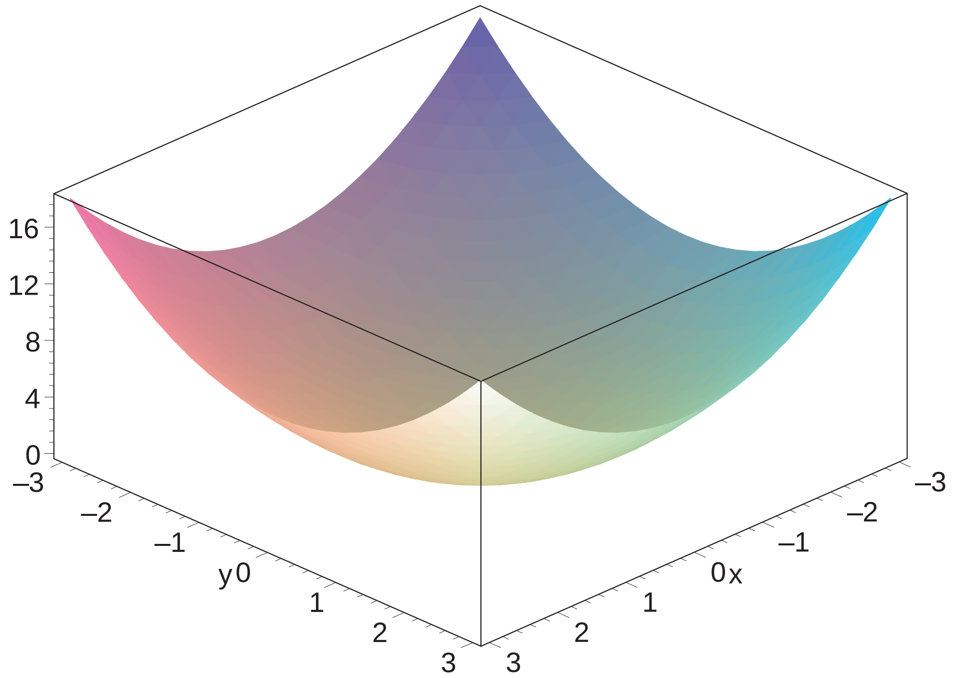
```
> simplify(sin(x)^2+cos(x)^2);
```

$$1$$

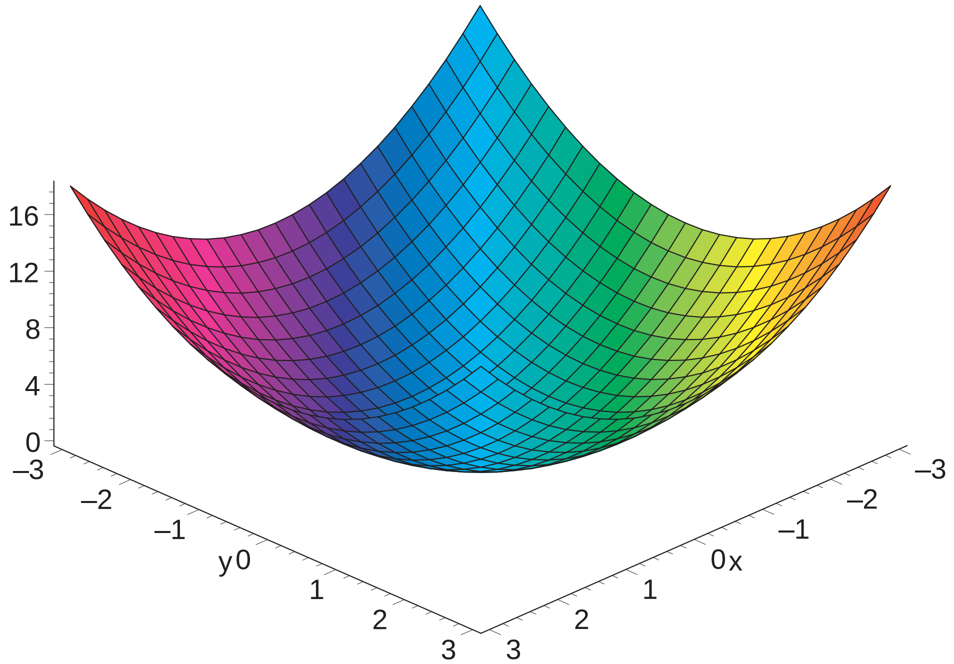
```
> ?plot
> p:=plot3d(x^2+y^2,x=-3..3,y=-3..3,axes=boxed):
> p;
```




```
> with(plots): display(p,style=patchnogrid);
```




```
> plot3d(x^2+y^2,x=-3..3,y=-3..3,axes=framed,color=x-y);
```




```
> fib(0):=1;
```

$$\mathrm{fib}(0) := 1$$

```
> fib(1):=1;
```

$$\mathrm{fib}(1) := 1$$

```
> fib(n):=fib(n-1)+fib(n-2);
```

$$\mathrm{fib}(n) := \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$$

```
> 6+fib(2);
```

$$6 + \mathrm{fib}(2)$$

```
> 6+fib(n);
```

$$6 + \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$$

```
> fib:=n->'fib(n-1)+fib(n-2)';
```

$$fib := n \to \text{'}\mathrm{fib}(n-1) + \mathrm{fib}(n-2)\text{'}$$

```
> fib(2);
```

$$\mathrm{fib}(1) + \mathrm{fib}(0)$$

```
> fib(67);
```

$$\mathrm{fib}(66) + \mathrm{fib}(65)$$

```
> fib(n+1);
```

$$\mathrm{fib}(n) + \mathrm{fib}(n-1)$$

```
> fibp:=proc(i::nonnegint)
  if i<=1 then 1
  else fibp(i-1)+fibp(i-2) fi
  end;
```

$$fibp := \mathbf{proc}(i::nonnegint)\ \mathbf{if}\ i \le 1\ \mathbf{then}\ 1\ \mathbf{else}\ \mathrm{fibp}(i-1) + \mathrm{fibp}(i-2)\ \mathbf{end\ if\ end\ proc}$$

Hier muss man mit STOP im Menue arbeiten:

```
> fibp(30);
Warning, computation interrupted
```

```
> fibp(10);
```

$$89$$

```
> fibp:=proc(x::nonnegint)
  option remember;
  if x*x=x then 1
  else fibp(x-1)+fibp(x-2) fi
  end;
```

$fibp :=$ **proc**$(x::nonnegint)$

option *remember*;

if x^2 $= x$ **then** 1 **else** $\mathrm{fibp}(x-1) + \mathrm{fibp}(x-2)$ **end if**

end proc

```
> fibp(30);
```

$$1346269$$

```
> fibp(10000);
```

5443837311356528133873426099375038013538918455469596702624771584120858\
865622349017083051547938960541173822675978026317384359584751116241439174\
702642959169925586334117906063048089793531476108466259072759367899150677\
960088306597966641965824937721800381441158841042480997984696487375337180\
028163763317781927941101369262750979509800713596718023814710669912644214\
775254478587674568963808002962265133111359929762726679441400101575800043\
510777465935805362502461707918059226414679005690752321895868142367849593\
880756423483754386342639635970733756260098962462668746112041739819404875\
062443709868654315626847186195620146126642232711815040367018825205314845\
875817193533529827837800351902529239517836689467661917953884712\

44102846393544948461445077876252952096188759727288922076853739647586954\
31591724345371936112637439263373130058961672480517379863063681150030883\
96749587102619524631352447499505204198305187168321623283859794627245919\
77145462821839969578922379891219943177546970521613108109655995063829726\
12538482420078971090547540284381496119304650618661701229832889643527337\
50792786069444761853525144421077928045979904561298129423809156055033032\
33891960916223669875992278292319189668801771857555520994653320128446 50\
23711537151417492909131048972034555775071966454252328620220195060914835\
85223882711016708433051169942115775151255510251655931888164048344129557\
03882547752111157739578011586839707260256561482495646053870028033131186\
14853998053970315557275296933995860798503815814462764338588285295358034\
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15068982393544219888054293324403711948672155435765485654991345192710989\
19802665184564927827827212957649240235507595558205647569365394873317659\
00020637312657064350970948264971003873351747771340331902810557566793178\
94700241188030946040343629534719974613922747915497303564126330742308240\
51999996101549784667340458326852960388301120765629245998136251652347093\
96304973404644510636530416363082366924225776146828846179184322479343440\
6079917883360676846711185597501

```
> evalf(%);
```

$$.5443837311\ 10^{2090}$$

```
> evalf(%%,60);
```

$$.544383731135652813387342609937503801353891845546959670262477\ 10^{2090}$$