Uebungen am 15.11.2001

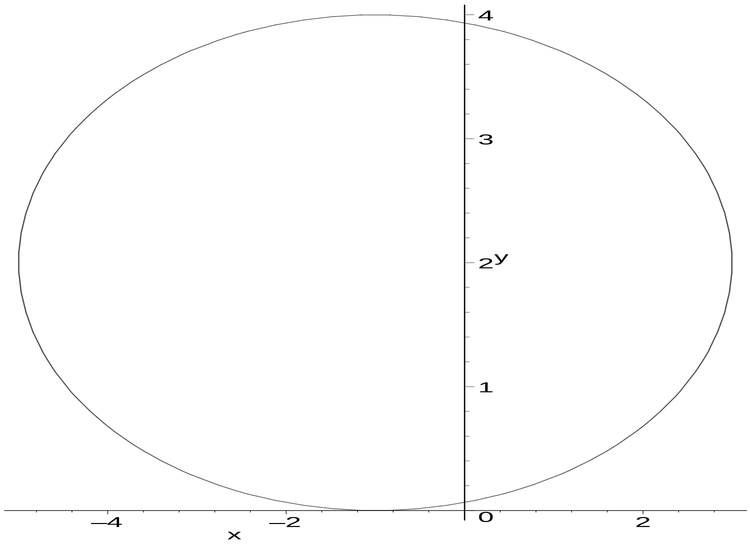
zu "Computergestuetztes symbolisches Rechnen" im Wintersemester 2001/02, Institut fuer Informatik der Universitaet Muenchen

1 Die Ellipse

```
> restart;
> ?implicitplot
> ellipse:=x^2+4*y^2+2*x-16*y+1=0;
```

$$\mathit{ellipse} := x^2 + 4\,y^2 + 2\,x - 16\,y + 1 = 0$$

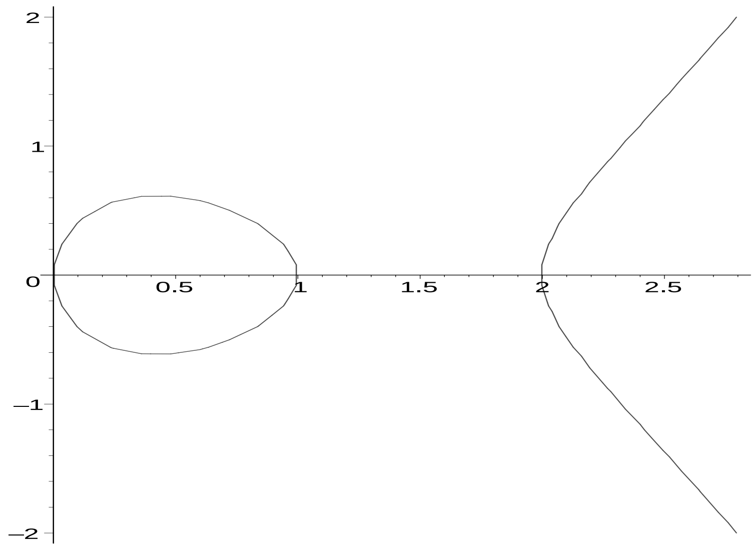
```
> with(plots):
Warning, the name changecoords has been redefined
> implicitplot(ellipse,x=-5..3,y=0..4);
```



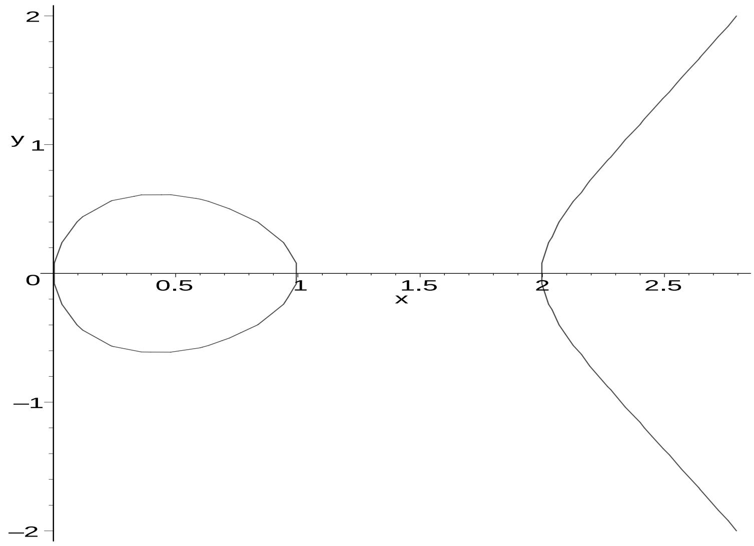

```
> funk2:=proc(x,y)y^2-x*(x-1)*(x-2)end proc;
```

$$\mathit{funk2} := \mathbf{proc}(x,\,y)\,y^2 - x * (x-1) * (x-2)\,\mathbf{end\ proc}$$

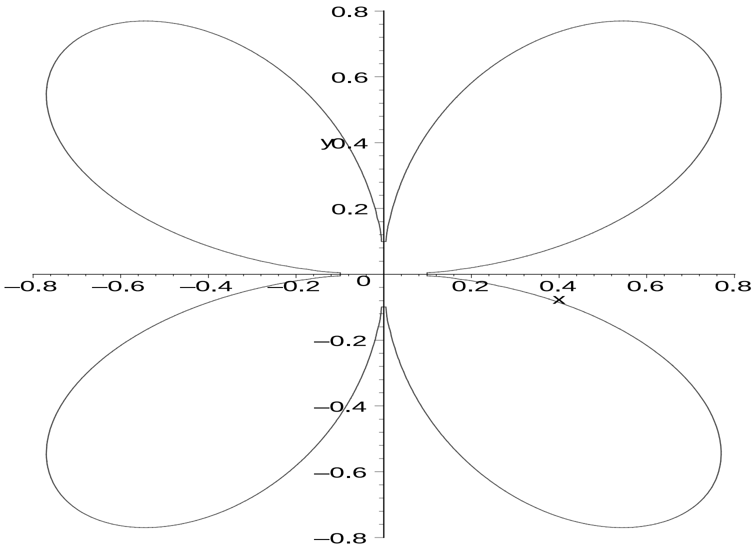
```
> implicitplot(funk2,0..3,-2..2);
```




```
>  implicitplot(y^2-x*(x-1)*(x-2)=0,x=0..3,y=-2..2);
```



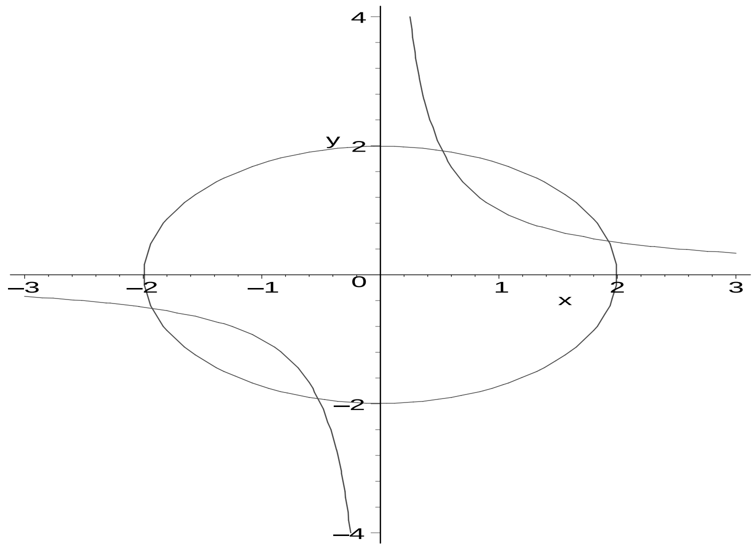

```
>  implicitplot((x^2+y^2)^3-4*x^2*y^2,x=-1..1,y=-1..1,
>  grid=[200,200]);
```




```
> ?solve
> solve((x^2+y^2)^3-4*x^2*y^2=0,x);
```

$$\frac{1}{3}\,\frac{\sqrt{-3\,\%1^{(1/3)}\,(-\%1^{(2/3)}-12+3\,y\,\%1^{(1/3)})\,y}}{\%1^{(1/3)}},$$
$$-\frac{1}{3}\,\frac{\sqrt{-3\,\%1^{(1/3)}\,(-\%1^{(2/3)}-12+3\,y\,\%1^{(1/3)})\,y}}{\%1^{(1/3)}},$$
$$\frac{1}{6}\,\frac{\sqrt{6}\,\sqrt{\%1^{(1/3)}\,(-\%1^{(2/3)}-12-6\,y\,\%1^{(1/3)}+I\,\sqrt{3}\,\%1^{(2/3)}-12\,I\,\sqrt{3})\,y}}{\%1^{(1/3)}},$$
$$-\frac{1}{6}\,\frac{\sqrt{6}\,\sqrt{\%1^{(1/3)}\,(-\%1^{(2/3)}-12-6\,y\,\%1^{(1/3)}+I\,\sqrt{3}\,\%1^{(2/3)}-12\,I\,\sqrt{3})\,y}}{\%1^{(1/3)}},$$
$$\frac{1}{6}\,\frac{\sqrt{-6\,\%1^{(1/3)}\,(\%1^{(2/3)}+12+6\,y\,\%1^{(1/3)}+I\,\sqrt{3}\,\%1^{(2/3)}-12\,I\,\sqrt{3})\,y}}{\%1^{(1/3)}},$$
$$-\frac{1}{6}\,\frac{\sqrt{-6\,\%1^{(1/3)}\,(\%1^{(2/3)}+12+6\,y\,\%1^{(1/3)}+I\,\sqrt{3}\,\%1^{(2/3)}-12\,I\,\sqrt{3})\,y}}{\%1^{(1/3)}}$$
$$\%1 := -54\,y+6\,\sqrt{-48+81\,y^2}$$

```
> implicitplot({x^2+y^2-4=0,x*y=1
> },x=-3..3,y=-4..4,scaling=constrained);
```




```
>  solve({x^4-z*x=0, x^3-y*x=0},{x,y});
```

$$\{x = 0, y = y\}, \{x = \mathrm{RootOf}(\_Z^3 - z, \mathit{label} = \_L2), y = \mathrm{RootOf}(\_Z^3 - z, \mathit{label} = \_L2)^2\}$$

```
>  loes:=%[2];
```

$$\mathit{loes} := \{x = \mathrm{RootOf}(\_Z^3 - z, \mathit{label} = \_L2), y = \mathrm{RootOf}(\_Z^3 - z, \mathit{label} = \_L2)^2\}$$

```
>  allvalues(eval(x,loes));
```

$$z^{(1/3)},\ z^{(1/3)}\,(-1)^{(2/3)},\ -z^{(1/3)}\,(-1)^{(1/3)}$$

```
>  groebnerbase({x^2-y^2-4,x*y},{x,y});
```

$$\mathrm{groebnerbase}(\{x\,y,\ x^2 - y^2 - 4\},\ \{y,\ x\})$$

```
>  ?groebne
>  ?Groebner
>  with(Groebner,gbasis);
```

$$[\mathit{gbasis}]$$

```
>  Basis:=gbasis({x^2+y^2-4,x*y-1},plex(x,y));
```

$$\mathit{Basis} := [1 + y^4 - 4\,y^2,\ x + y^3 - 4\,y]$$

```
>  Basis1:=gbasis({x^2+y^2-4,x*y-1},plex(y,x));
```

$$\mathit{Basis1} := [1 + x^4 - 4\,x^2,\ y + x^3 - 4\,x]$$

```
>  with(Groebner);
```

[*fglm*, *gbasis*, *gsolve*, *hilbertdim*, *hilbertpoly*, *hilbertseries*, *inter_reduce*, *is_finite*, *is_solvable*, *leadcoeff*, *leadmon*, *leadterm*, *normalf*, *pretend_gbasis*, *reduce*, *spoly*, *termorder*, *testorder*, *univpoly*]

```
>  testorder(x^2*y,x*y^2,plex(x,y));
```

$$\mathit{false}$$

```
> testorder(x^2*y,x*y^2,plex(y,x));
```
$$true$$
```
> normalf(x^2+y^2-4,Basis1,plex(x,y));
```
$$x^2+y^2-4$$
```
> normalf(x^2+y^2-4,Basis,plex(x,y));
```
$$0$$
```
> loesungen:=solve(Basis[1],y);
```
$$loesungen := -\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ -\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ \frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}$$
```
> expand(simplify(loesungen[3]^2));
```
$$2-\sqrt{3}$$
```
> ?eval
> eval(4*y-y^3,y=loesungen[3]);
```
$$-(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2})^3-2\sqrt{6}+2\sqrt{2}$$
```
> simplify(%);
```
$$-\frac{1}{2}\sqrt{2}\sqrt{3}-\frac{1}{2}\sqrt{2}$$
```
> %+(2+sqrt(3))*sqrt(2-sqrt(3));
```
$$-\frac{1}{2}\sqrt{2}\sqrt{3}-\frac{1}{2}\sqrt{2}+(2+\sqrt{3})\,(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2})$$
```
> expand(%);
```
$$-\sqrt{2}\sqrt{3}-\frac{3}{2}\sqrt{2}+\sqrt{6}+\frac{1}{2}\sqrt{3}\sqrt{6}$$
```
> simplify(%);
```
$$0$$
```
> solve({x^2+y^2-4=0,x*y=1},{x,y});
```
$$\{y=\mathrm{RootOf}(1+\_Z^4-4\,\_Z^2),$$
$$x=-\mathrm{RootOf}(1+\_Z^4-4\,\_Z^2)^3+4\,\mathrm{RootOf}(1+\_Z^4-4\,\_Z^2)\}$$
```
> allvalues(%);
```
$$\{y=-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ x=-(-\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2})^3-2\sqrt{6}-2\sqrt{2}\},$$
$$\{y=\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ x=-(\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2})^3+2\sqrt{6}+2\sqrt{2}\},$$
$$\{y=-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2},\ x=-(-\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2})^3-2\sqrt{6}+2\sqrt{2}\},$$
$$\{y=\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2},\ x=-(\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2})^3+2\sqrt{6}-2\sqrt{2}\}$$
```
> ?polynomial
> type(x*y-5*x^3/y,polynom);
```

false

```
>  type(x*y-5*x^3/x,polynom);
```

true

```
>  type(sin(x)^2+cos(x)^2,polynom);
```

false

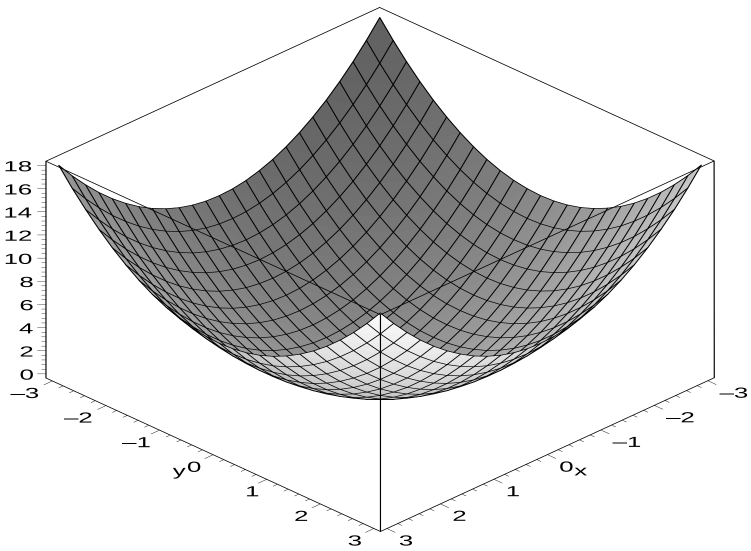
```
>  type(simplify(sin(x)^2+cos(x)^2),polynom);
```

true

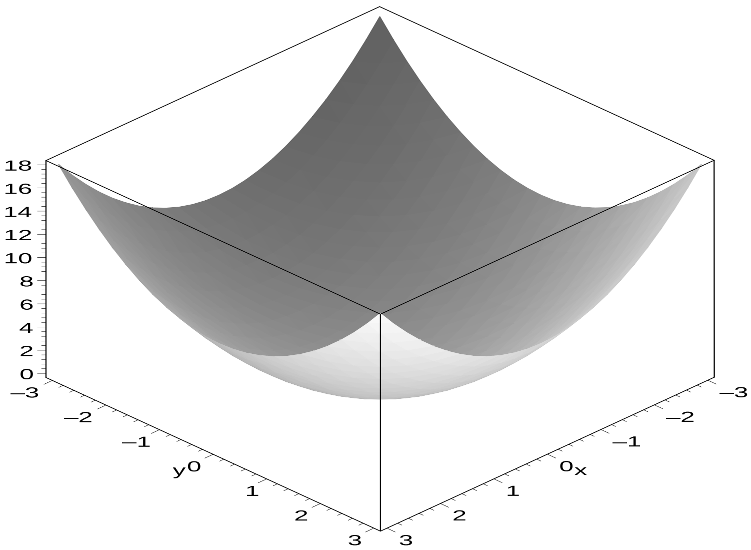
```
>  simplify(sin(x)^2+cos(x)^2);
```

1

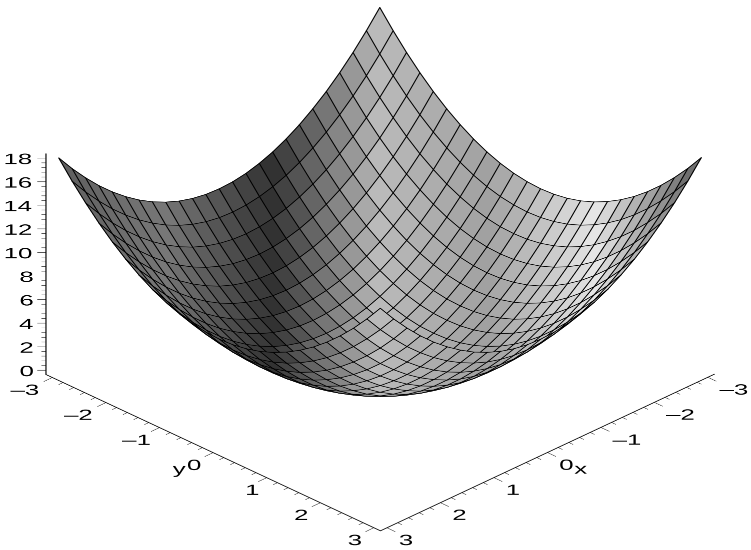
```
>  ?plot
>  p:=plot3d(x^2+y^2,x=-3..3,y=-3..3,axes=boxed):
>  p;
```




```
>  with(plots):  display(p,style=patchnogrid);
```




```
> plot3d(x^2+y^2,x=-3..3,y=-3..3,axes=framed,color=x-y);
```




```
> fib(0):=1;
```

$$\mathrm{fib}(0) := 1$$

```
> fib(1):=1;
```

$$\mathrm{fib}(1) := 1$$

```
> fib(n):=fib(n-1)+fib(n-2);
```

$$\mathrm{fib}(n) := \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$$

```
> 6+fib(2);
```

$$6 + \mathrm{fib}(2)$$

```
> 6+fib(n);
```

$$6 + \mathrm{fib}(n-1) + \mathrm{fib}(n-2)$$

```
> fib:=n->'fib(n-1)+fib(n-2)';
```

$$\mathit{fib} := n \to \text{'}\mathrm{fib}(n-1) + \mathrm{fib}(n-2)\text{'}$$

```
> fib(2);
```

$$\mathrm{fib}(1) + \mathrm{fib}(0)$$

```
> fib(67);
```

$$\mathrm{fib}(66) + \mathrm{fib}(65)$$

```
> fib(n+1);
```

$$\mathrm{fib}(n) + \mathrm{fib}(n-1)$$

```
> fibp:=proc(i::nonnegint)
> if i<=1 then 1
> else fibp(i-1)+fibp(i-2) fi
> end;
```

$$\mathit{fibp} := \mathbf{proc}(i{::}\mathit{nonnegint})\,\mathbf{if}\,i \le 1\,\mathbf{then}\,1\,\mathbf{else}\,\mathrm{fibp}(i-1) + \mathrm{fibp}(i-2)\,\mathbf{end\ if}\,\mathbf{end\ proc}$$

Hier muss man mit STOP im Menue arbeiten:

```
> fibp(30);

Warning, computation interrupted

> fibp(10);
```

$$89$$

```
> fibp:=proc(x::nonnegint)
> option remember;
> if x*x=x then 1
> else fibp(x-1)+fibp(x-2) fi
> end;
```

$$\begin{aligned}
&\mathit{fibp} := \mathbf{proc}(x{::}\mathit{nonnegint})\\
&\mathbf{option}\ \mathit{remember};\\
&\quad \mathbf{if}\,x^2 = x\,\mathbf{then}\,1\,\mathbf{else}\,\mathrm{fibp}(x-1) + \mathrm{fibp}(x-2)\,\mathbf{end\ if}\\
&\mathbf{end\ proc}
\end{aligned}$$

```
> fibp(30);
```

$$1346269$$

```
> fibp(10000);
```

54438373113565281338734260993750380135389184554695967026247715841\
20858286562234901708305154793896054117382267597802631738435\
95847511162414391747026429591699255863341179060630480897935\
31476108466259072759367899150677960088306597966641965824937\
72180038144115884104248099798469648737533718002816376331778\
19279411013692627509795098007135967180238147106699126442147\
75254478587674568963808002962265133111359929762726679441400\
10157580004351077746593580536250246170791805922641467900569\
07523218958681423678495938807564234837543863426396359707337\
56260098962462668746112041739819404875062443709868654315626\
84718619562014612664223271181504036701882520531484587581719\
35335298278378003519025292395178366894676619179538847124410\
28463935449484614450778762529520961887597272889220768537396\
47586954315917243453719361126374392633731300589616724805173\
79863063681150030883967495871026195246313524474995052041983\
05187168321623283859794627245919771454628218399695789223798\
91219943177546970521613108109655995063829726125384824200789\
71090547540284381496119304650618661701229832889643527337507\
92786069444761853525144421077928045979904561298129423809156\
05503303233891960916223669875992278292319189668801771857555\
55209946533201284465023711537151417492909131048972034555775\
07196645425232862022019506091483585223882711016708433051169\
94211577515125551025165593188816404834412955703882547752111\
15773957801158683970726025656148249564605387002803313118614\
85399805397031555727529693399586079850381581446276433858828\
52953580342485084542644647168153100153318047956743639681565\
33261525095711274804119281960221488491482843891241785201745\
07305538928717857923509417743383331506898239354421988805429\
33244037119486721554357654856549913451927109891980266518456\
49278278272129576492402355075955582056475693653948733176590\
00206373126570643509709482649710038733517477713403319028105\
57566793178947002411880309460403436295347199746139227479154\
97303564126330742308240519999961015497846673404583268529603\
88301120765629245998136251652347093963049734046445106365304\
16363082366924225776146828846179184322479343440607991788336\
0676846711185597501

```
>  evalf(%);
```

$$.5443837311\ 10^{2090}$$

```
> evalf(%%,60);
```

$$.544383731135652813387342609937503801353891845546959670262477\, 10^{2090}$$