Aufgabe 1

```
> restart;
> n:=10;
```

$$n := 10$$

```
> q:=quo(x^n-a^n,x-a,x);
```

$$q := x^9 + a\,x^8 + a^2\,x^7 + a^3\,x^6 + a^4\,x^5 + a^5\,x^4 + a^6\,x^3 + a^7\,x^2 + a^8\,x + a^9$$

```
> q:=n->quo(x^n-a^n,x-a,x);
```

$$q := n \to \mathrm{quo}(x^n - a^n, x - a, x)$$

```
> q(10);
```

$$x^9 + a\,x^8 + a^2\,x^7 + a^3\,x^6 + a^4\,x^5 + a^5\,x^4 + a^6\,x^3 + a^7\,x^2 + a^8\,x + a^9$$

```
> q(4);
```

$$x^3 + a\,x^2 + a^2\,x + a^3$$

```
> q(1);
```

$$1$$

Aufgabe 2, erstes Problem

```
> f := x^4-6*x^2+12*x-8;
```

$$f := x^4 - 6\,x^2 + 12\,x - 8$$

```
> g := 2*x^3 -10*x^2 +16*x -8;
```

$$g := 2\,x^3 - 10\,x^2 + 16\,x - 8$$

```
> q:=quo(f,g,x,r);
```

$$q := \frac{1}{2}\,x + \frac{5}{2}$$

```
> r;
```

$$12 + 11\,x^2 - 24\,x$$

```
> q1:=quo(g,r,x,r1);
```

$$q1 := \frac{2}{11}\,x - \frac{62}{121}$$

```
> r1;
```

$$-\frac{224}{121} + \frac{184}{121}\,x$$

```
> q2:=quo(r,r1,x,r2);
```

$$q2 := \frac{1331}{184}\,x - \frac{7381}{1058}$$

```
> r2;
```

$$\frac{-484}{529}$$

```
> q3:=quo(r1,r2,x,r3);
```

$$q3 := \frac{29624}{14641} - \frac{24334}{14641}\,x$$

```
> r3;
```

$$0$$

Also ist r2 ein ggT. Dann aber auch 1:

```
gcd(f,g);
```

$$1$$

```
> ?gcdex
> gcdex(f,g,x,a,b);
```

$$1$$

```
> a;
```

$$-5+\frac{87}{16}x-\frac{23}{16}x^2$$

```
> b;
```

$$\frac{39}{8}-\frac{51}{16}x+\frac{7}{8}x^2+\frac{23}{32}x^3$$

```
> a*f+b*g;
```

$$\left(-5+\frac{87}{16}x-\frac{23}{16}x^2\right)(x^4-6x^2+12x-8)+\left(\frac{39}{8}-\frac{51}{16}x+\frac{7}{8}x^2+\frac{23}{32}x^3\right)(2x^3-10x^2+16x-8)$$

```
> simplify(%);
```

$$1$$

Aufgabe 2, zweites Problem

```
> f := x^9 -1;
```

$$f := x^9-1$$

```
> g := x^5 +x^3 -x^2 -1;
```

$$g := x^5+x^3-x^2-1$$

```
> gg := gcd(f,g);
```

$$gg := x^3-1$$

```
> gcdex(f,g,x,a,b);
Error, (in gcdex/polynom) illegal use of a formal parameter
> gcdex(f,g,x,'a','b');
```

$$x^3-1$$

```
> a;
```

$$x$$

```
> b;
```

$$1-x-x^5+x^3-x^2$$

```
> simplify(a*f+g*b);
```

$$x^3-1$$

Aufgabe 2, drittes Problem

```
> f1 := x^3 + x^2 -4*x -4;
```

$$f1 := x^3+x^2-4x-4$$

```
> f2 := x^3 - x^2 - 4*x +4 ;
```

$$f2 := x^3-x^2-4x+4$$

```
> gg:=gcd(f1,f2);
```

$$gg := -4+x^2$$

Nun nur aus Interesse:

```
> f3 := x^3 -2*x^2 -x + 2;
```

$$f3 := x^3-2x^2-x+2$$

```
> gcd(gg,f3);
```

$$x-2$$

Aufgabe 3

```
> f := x^5 -2*x^4 + 2*x^2 -x ;
```

$$f := x^5 - 2x^4 + 2x^2 - x$$

```
> g := x^5 - x^4 -2*x^3 +2*x^2 +x -1;
```

$$g := x^5 - x^4 - 2x^3 + 2x^2 + x - 1$$

```
> solve({f,g},x);
```

$$\{x = -1\}, \{x = 1\}$$

Wie kann man das "zu Fuss" einsehen?

```
> f-g;
```

$$-x^4 - 2x + 2x^3 + 1$$

Rate die Nullstelle 1

```
> eval(f-g,x=1);
```

$$0$$

```
> eval(f,x=1);
```

$$0$$

```
> eval(g,x=1);
```

$$0$$

```
> f1:=quo(f,x-1,x);
```

$$f1 := x^4 - x^3 - x^2 + x$$

```
> g1:=quo(g,x-1,x);
```

$$g1 := x^4 - 2x^2 + 1$$

Die Nullstellen von g1 findet man als Loesungen einer biquadratischen Gleichung:

```
> solve(y^2-2*y+1,y);
```

$$1, 1$$

Also nur -1 und 1 zu pruefen.

```
> eval(f1,x=-1);
```

$$0$$

```
> eval(f1,x=1);
```

$$0$$