Übungen am 29.11.2001

Übungen zu "Computergestütztes symbolisches Rechnen" im Wintersemester 2001/02, Institut für Informatik der Universität München

■ Zu Aufgabe 5, 1. und 2.

```
f = x^7 y^2 + x^3 y^2 - y + 1
```

$1 - y + x^3 y^2 + x^7 y^2$

```
gg = {x y^2 - x , x - y^3}
```

$\{-x + x y^2, x - y^3\}$

```
gg1 = Reverse[gg]
```

$\{x - y^3, -x + x y^2\}$

```
? PolynomialReduce
```

```
PolynomialReduce[poly, {poly1, poly2, ... }, {x1, x2, ... }] yields a list representing a
   reduction of poly in terms of the polyi. The list has the form {{a1, a2, ... },
   b}, where b is minimal and a1 poly1 + a2 poly2 + ...  + b is exactly poly. More...
```

```
PolynomialReduce[f, gg, {x, y}, MonomialOrder → Lexicographic]
```

$\{\{2 + 2 x^2 + x^4 + x^6 + 2 x y + x^3 y + x^5 y + 2 y^2 + x^2 y^2 + x^4 y^2,$
$2 + 2 x^2 + x^4 + x^6 + 2 x y + x^3 y + x^5 y\}, 1 - y + 2 y^3\}$

```
Part[%, 2]
```

$1 - y + 2 y^3$

```
PolynomialReduce[f, gg1, {x, y}, MonomialOrder → Lexicographic]
```

$\{\{x^2 y^2 + x^6 y^2 + x y^5 + x^5 y^5 + y^8 + x^4 y^8 + x^3 y^{11} + x^2 y^{14} + x y^{17} + y^{20}, 0\}, 1 - y + y^{11} + y^{23}\}$

```
Part[%, 2]
```

$1 - y + y^{11} + y^{23}$

```
PolynomialReduce[f, gg, {x, y}, MonomialOrder → DegreeLexicographic]
```

$\{\{x^2 + x^6, 0\}, 1 + x^3 + x^7 - y\}$

```
Last[%]
```

$1 + x^3 + x^7 - y$

```
PolynomialReduce[f, gg1, {x, y}, MonomialOrder → DegreeLexicographic]
```

$\{\{0, x^2 + x^6\}, 1 + x^3 + x^7 - y\}$

■ Zu Aufgabe 5, 3. und 4.

```
f = x y^2 z^2 + x y - y z
```

$$x\,y - y\,z + x\,y^2\,z^2$$

```
g1 = x - y^2
```

$$x - y^2$$

```
g2 = y - z^3
```

$$y - z^3$$

```
g3 = z^2 - 1
```

$$-1 + z^2$$

```
g123 = {g1, g2, g3}
```

$$\{x - y^2,\ y - z^3,\ -1 + z^2\}$$

```
PolynomialReduce[f, g123, {x, y, z}, MonomialOrder → Lexicographic]
```

$$\{\{y + y^2\,z^2,\ y^2 - z + y^3\,z^2 + y\,z^3 + y^2\,z^5 + z^6 + y\,z^8 + z^{11},\\ z + z^3 + z^4 + z^5 + z^6 + z^7 + z^8 + z^{10} + z^{12}\},\ z\}$$

Aus Interesse sehen wir uns noch die Situation für grlex an.

```
PolynomialReduce[f, g123, {x, y, z}, MonomialOrder → DegreeLexicographic]
```

$$\{\{-x\,z^2,\ 0,\ x^2\},\ x^2 + x\,y - y\,z\}$$

```
PolynomialReduce[f, {g2, g3, g1}, {x, y, z}, MonomialOrder → Lexicographic]
```

$$\{\{x + y - z + y\,z + x\,y\,z^2 + z^3 + z^4 + x\,z^5,\ x + z + x\,z + x\,z^2 + z^3 + z^4 + x\,z^4 + z^5 + x\,z^6,\ 1 + z\},\ z\}$$

```
PolynomialReduce[f, {g2, g3, g1}, {x, y, z}, MonomialOrder → DegreeLexicographic]
```

$$\{\{0,\ x\,y^2,\ -x\},\ x^2 + x\,y - y\,z\}$$

```
PolynomialReduce[f, {g3, g1, g2}, {x, y, z}, MonomialOrder → Lexicographic]
```

$$\{\{y + y^2 + x\,y^2 + z + y\,z + y^2\,z + y^3\,z + y\,z^2 + y^2\,z^2,\ y + y^2,\ 1 + y + y^2 + y^3 + y\,z + y^2\,z\},\ z\}$$

```
PolynomialReduce[f, {g3, g1, g2}, {x, y, z}, MonomialOrder → DegreeLexicographic]
```

$$\{\{x\,y^2,\ -x,\ 0\},\ x^2 + x\,y - y\,z\}$$

■ Zu Aufgabe 6

```
g1 = 2 x y^2 - x
```

$$-x + 2\,x\,y^2$$

```
g2 = 3 x^2 y - y - 1
```

$$-1 - y + 3\,x^2\,y$$

f wird natürlich so gewählt, daß sich die Leitterme auslöschen.

```
f = 3 x * g1 - 2 y * g2
```

```
-2 y (-1 - y + 3 x^2 y) + 3 x (-x + 2 x y^2)
```

```
Expand[%]
```

```
-3 x^2 + 2 y + 2 y^2
```

```
PolynomialReduce[f, {g1, g2}, {x, y}, MonomialOrder → DegreeLexicographic]
```

```
{{0, 0}, -3 x^2 + 2 y + 2 y^2}
```

Wir untersuchen aus Interesse noch eine Abwandlung von g1.

```
g1 = 2 x y^2 - x y
```

```
-x y + 2 x y^2
```

```
f = 3 x * g1 - 2 y * g2
```

```
-2 y (-1 - y + 3 x^2 y) + 3 x (-x y + 2 x y^2)
```

```
Expand[%]
```

```
2 y - 3 x^2 y + 2 y^2
```

```
PolynomialReduce[f, {g1, g2}, {x, y}, MonomialOrder → DegreeLexicographic]
```

```
{{0, -1}, -1 + y + 2 y^2}
```

Hier erhalten wir wieder einen Rest ungleich Null, aber nicht f als Rest.